ABV- Indian Institute of Information Technology & Management, Gwalior

Semester I (2023-24)

Major

Course Title: Engineering Mathematics-I

Course Code: ES102

MM: 60 – 65 **Duration: 180 minutes**

Note:

1. All parts of a question should be answered consecutively.
2. The question paper has six questions in two pages.
3. Questions no. 1 b) and 5 b) are open questions, and the marks will be purely based on the justification and Mathematical explanations.
4. Question No. 3 c), d), 4 a): Marks are only for the proper justification.
5. Question no. 3 b): you must show that the example is a commutative ring with identity but not a field

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1. a) Solve the following system using LU-Decomposition
$2x_1 + 3x_2 + x_3 = 9;\ x_1 + 2x_2 + 3x_3 = 6; 3x_1 + x_2 + 2x_3 = 8$ **(5)**
b) How can we construct symmetric and skew symmetric matrix from any random real square matrix **(2-5)**
c) State Cayley Hamilton Theorem (CHT) **(1)**
d) verify CHT for the following matrix using CHT and also find its inverse using CHT
$$\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$$ **(6)**

2. a) Transform the following quadratic for to canonical form
$3x_1^2 - 2x_2^2 - x_3^2 - 4x_1x_2 + 12x_2x_3 + 8x_1x_3$ **(7)**
b) Fill in the blanks and prove: The _____ vectors of a _____ matrix forms a unitary system. **(3)**

on b/w 2 vector spaces.

3. a) Define Isomorphism ~~between two~~ linear transformation. **(2)**
b) Give an example of a finite commutative ring with identity but not a field and prove your claim. **(2.5)**
c) Is the vector $(0,4,-4,2)$ in the subspace of R^4 spanned by the vectors $(1,1,1,1), (1,1,1,-1), (1,1,-1,1), (1,-1,1,1)$? **(2.5)**
d) Find out the vector in R^4 which is not in the subspace spanned by the vectors $(1,1,1,1), (1,1,1,-1), (1,1,-1,1), (1,-1,1,1)$ **(2.5)**

4. a) Which of the following are linear transformations from R^2 to R^2 **(2)**
$i)\ T(x,y) = (x + 11, y + 23), ii)\ T(x,y) = (x + 11y, 0), iii)\ T(x,y) = (x^2, y)$

b) Let F be a subfield of the complex numbers and let T be the function from F^3 *to* F^3 defined by $T(x,y,z) = (x - y + 2z, 2x + y, -x - 2y + 2z)$

i) check whether T is a Linear Transformation

ii) If (a, b, c) is a vector in F^3, what are the conditions on a, b, c that the vector be in the range of T?

iii) what are the conditions on a, b, c that the vector (a, b, c) be in the nullspace of T? **(2+1.5+1.5)**

5. a) Find the spanning set for the null space of the matrix

$$\begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$$ **(3)**

b) Define Vector space, Construct your own example of a Vector space and discuss any one subspace of the identified vector space. **(4.5-6.5)**

6. a) Find $\lim_{(x,y)\to(1,2)}(x^2y^3) - x^3y^2 + 3x + 2y)$ **(2)**

b) Find out $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for the following function using limiting conditions (limit definition of derivative) $f(x,y) = 17x^2 + 21xy - 4y^2 + 13x - 23y + 51$ **(1.5+1.5)**

c) Find out the absolute maximum and absolute minimum for the following,

$f(x,y) = x^2 - 2xy + 4y^2 - 4x - 2y + 24, where\ 0 \le x \le 5\ and\ 0 \le y \le 3$ **(7)**

*****End of Question Paper*****
*****Best of Luck*****