[ST] Enrol. No. ....

END SEMESTER EXAMINATION : JAN. 2022

APPLIED MATHEMATICS – I

Time : 3 Hrs. *Maximum Marks : 60*

Note: ***Attempt questions from all sections as directed. nonprogrammable scientific calculator is permitted.***

SECTION – A (24 Marks)

Attempt any **four** *questions out of* **five**.

Each question carries 06 *marks.*

1. Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

2. Solve the system of equation using Gauss Jordon method :

$$x + y + z = 3;$$

$$x + 2y + 3z = 4;$$

$$x + 4y + 9z = 6.$$

P.T.O.

3. Change the order of the integration and hence evaluate

$$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dydx}{\sqrt{y^4 - a^2x^2}}$$

4. Find the nth derivative of $\frac{x^2}{(x-1)^3(x-2)}$.

5. Show that $\bar{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find scalar ϕ such that $\bar{F} = \bar{\nabla}\phi$.

SECTION – B **(20 Marks)**

Attempt any **two** *questions out of* **three**.

Each question carries **10** *marks.*

6. Evaluate $\iint_S \left(x\hat{i} + y\hat{j} + z^2\hat{k}\right) \hat{n}\, dS$

where S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$.

7. (a) Find the area of the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the straight line $2x + 3y = 6$.

(6)

(b) For which value of 'b', the rank of the matrix

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix} \text{ is } 2.$$

(4)

8. (a) If $z = x^3 + x^2y + y^3$, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$.

(5)

(b) Expand logSinx in powers of $(x - 2)$ by Taylor's theorem.

(5)

SECTION - C (16 Marks)

(Compulsory)

9. (a) Using Cayley Hamilton theorem, find A^{-1} where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$ Also find the matrix

represented by the expression

$A^6 - 3A^5 + A^4 - 2A^3 - 3A^2 + A - 2I.$ (8)

(b) Find the n^{th} derivative of $\sin^2 x.\cos^3 x$. (8)