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MATH114 Enrol. No.

[ST]

END SEMESTER EXAMINATIONS JANUARY 2025

APPLIED MATHEMATICS- I

Time : 3 Hrs. *Maximum Marks : 60*

Note: ***Attempt questions from all sections as directed. Non Programmable scientific calculator is permitted.***

SECTION – A (24 Marks)

Attempt any ***Four*** *questions out of* ***Five.***

Each question carries **06** *marks.*

1. Find the rank of the following matrix by reducing it into normal form:

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

2. If $u = \sin^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ Prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u\cos 2u}{4\cos^3 u}.$$

3. Evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}$ dx dy by changing the order of integration.

4. Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).

5. Solve, with the help of matrices, the simultaneous equations $3x + y + 2z = 3$, $2x-3y-z=-3$, $x + 2y + z = 4$

SECTION – B **(20 Marks)**

Attempt any **two** *questions out of* **three**.

Each question carries **10** *marks.*

6. (a) Expand e^{xy}at (1,1) using Taylor's theorem. (5)

(b) Find, by double integration, the smaller of the areas bounded by the circle (5)

$x^2 + y^2 = 9$ and the line $x + y = 3$.

7. State Gauss Divergence Theorem and use to evaluate

Fîn dS where $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the cylindrical region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

8. Find the eigen values and eigen vectors of matrix

$$A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}.$$

SECTION – C **(16 Marks)**

(Compulsory)

9. (a) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the maximum temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. (10)

(b) Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz.$ (6)