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MATH122 Enrol. No. ....

[ST]

END SEMESTER EXAMINATION : MAY, 2025

APPLIED MATHEMATICS – II

Time : 3 Hrs. *Maximum Marks : 60*

Note: ***Attempt questions from all sections as directed. Use of Scientific Calculator is allowed.***

SECTION – A (24 Marks)

Attempt any **four** *questions out of* **five**. *Each question carries* **06** *marks.*

1. Solve the differential equation

$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0.$

2. Find the inverse Laplace transform of $\log \frac{s^2 - 1}{s^2}$.

3. Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also express f(z) in terms of z.

P.T.O.

4. Solve the differential equation

$$(2x \log x - xy)\, dy + 2y\, dx = 0$$

5. Evaluate the integral $\int_0^{1+i} \left(x^2 - iy\right) dz$ along the path

(i) $y = x$

(ii) $y = x^2$.

SECTION – B **(20 Marks)**

Attempt any **two** *questions out of* **three.**

Each question carries **10** *marks.*

6. (a) Obtain solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y - 6e^{3x} = 7e^{2x} - \log 2 \qquad (6)$$

(b) Evaluate the integral

$$\int_0^{\infty} e^{-t} \frac{\sin t}{t}\, dt \qquad (4)$$

7. (a) Using Laplace transform find the solution of the initial value of problem

$$\frac{d^2y}{dt^2} + 25y = 10\cos 5t;\; y(0) = 2,\; y'(0) = 0. \qquad (6)$$

(b) Show that the function f(z) defined by

$$f(z)=\begin{cases}\dfrac{\mathrm{Re}(z)}{z} & z \neq 0 \\ 0 & z=0\end{cases}$$

is not continuous at z = 0. (4)

8. (a) Define unit step function. Express the following function in terms of units step functions and hence find its Laplace transform:

$$f(t)=\begin{cases} t-1 & 1<t<2 \\ 3-t & 2<t<3 \end{cases}$$ (6)

(b) Find the residue of the function $f(z)=\dfrac{z^2}{(z+1)^2(z-2)}$ at its double pole. (4)

SECTION – C (16 Marks)

(Compulsory)

9. (a) Define Analytic function. Determine whether the function $f(z)=\dfrac{1}{z}$ is analytic or not. (6)

(b) Evaluate the integral $\oint_C \frac{dz}{z^2+9}$, where C is the curve

(i) $|z + 3i| = 2$

(ii) $|z| = 5$. (6)

(c) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = \sin 2x. \quad (4)$$