

**PART-A (10 X 2 = 20)**  
**(Answer all the questions)**

1 Find  $\frac{du}{dt}$  if  $u = x^2 + y^2, x = at^2, y = 2at$ .

2 State Maclaurin's series.

3 Change the order of integration  $\int_0^1 \int_0^x f(x, y) dy dx$

4 State Green's Theorem.

5 Evaluate:  $(D^4 - 1)y = 0$

6 Solve  $(\cos x - x \cos y) dy - (\sin y + y \sin x) dx = 0$

7 The product of two eigen values of the matrix  $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$  is 16. Find the third eigen value.

8	Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
9	Define Vector space.
10	Define Linear Span

**PART- B (5 X 16 = 80)**

11	(a) (i) Find the intervals on which $f$ is increasing or decreasing, local maximum or minimum, intervals of concavity and the inflection points: $f(x) = 2x^3 + 3x^2 - 36x$ .
	(a) (ii) Determine $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$
	(OR)
	(b) (i) Expand $e^x \log(1 + y)$ in powers of $x$ and $y$ using Taylor's series up to third degree terms.
12	(b) (ii) Examine the extreme values $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ .
	(a) (i) Verify Gauss's divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .
	(a) (ii) Show that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational vector
	(OR)
13	(b) (i) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$
	(b) (ii) By changing Cartesian to polar co-ordinates, show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy = \frac{\pi}{4}$ and hence evaluate $\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$
	(a) (i) Produce a complete solution for $(D^2 + a^2)y = \tan ax$
	(a) (ii) Find the singular integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$

	(b) (i) Convert $[(3x + 2)^2 D^2 + 3(3x + 2)D - 36]y = 3x^2 + 4x + 1$ into constant coefficients also produce a general solution.	CO3
	(b) (ii) Generate complete solution for the differential equation $(D^2 + 4)y = x^2 \cos 2x$	CO3
	(a) (i) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 + 4x_1x_3 - 2x_2x_3$ to canonical form through an orthogonal transformation and find its nature, rank and signature.	CO4
	(OR)	
14	(b) (i) Factorize the following system of equations by LU decomposition method $7x_1 - 2x_2 + x_3 = 12$ , $14x_1 - 7x_2 - 3x_3 = 17$ , $-7x_1 + 11x_2 + 18x_3 = 5$	CO
	(b) (ii) Solve the following system of equations by Gauss Jordan method $x + 2y + z = 3$ , $2x + 3y + 3z = 10$ , $3x - y + 2z = 13$	CO
	(a) (i) Let $V$ be the set of all positive real numbers. Define the vector addition and scalar multiplication as follows: $x + y = xy$ & $kx = x^k$ . Determine whether or not $V$ is a vector space over $F$ with respect to above operations.	CO
	(a) (ii) Let $V = R^3$ ; $W = \{(a_1, a_2, a_3) / 2a_1 - 7a_2 + a_3 = 0\}$ . Verify whether it is a subspace or not.	CO
15	(OR)	
	(b) (i) State and prove Dimension theorem (or) Rank-Nullity theorem	CO
	(b) (ii) Let $T: R^2 \rightarrow R^3$ and $U: R^2 \rightarrow R^3$ be the linear transformations respectively defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ , $U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$ , then prove that $[T+U]_{\beta}^{\gamma} = [T]_{\beta}^{\gamma} + [U]_{\beta}^{\gamma}$	CO5