(Please write your Enrollment Number) Enrollment No.____________

MID-TERM EXAMINATION

(Course Name :BTech CSAI/ECEAI/AI&ML) (Semester : II)

(May, 2023) OFF LINE mode

Subject Code: BAS 108	**Subject: Probability and Statistics**
Time :1 ½Hours	**Maximum Marks :** 30
Note:Q. 1 is compulsory.	

Q1 (2.5*4)

(a) The value of a piece of factory equipment after 3 years of use is $100(0.5)^X$ where X is a random variable having moment generating function $M_X(t) = \frac{1}{1-2t}$ for $t < 1/2$. Calculate the expected value of this piece of equipment after 3 years of use.

(b) Prove that, if A and B are independent events then A^c and B^c are also independent events.

(c) Prove that $\mathrm{Cov}(X_1 + X_2, Y) = \mathrm{Cov}(X_1, Y) + \mathrm{Cov}(X_2, Y)$.

(d) Prove or disprove, if $E(XY) = E(X).E(Y)$ then X and Y areindependent.

Q2 (Attempt any Two Parts) (5,5)

(a) Consider a random variable X with probability density function

$$f_X(x) = \begin{cases} 4x^3 & , if\ 0 < x < 1 \\ 0, & otherwise \end{cases}$$

Find Var(X).

(b) Let X be a continuous random variable with probability density function given by $f_X(x) = \frac{1}{2}e^{-|x|}\ \forall\ x\ \varepsilon\ R$. If Y = X^2, Find the Cumulative distribution function of Y.

(c) It is reported that 50% of all computer chips produced are defective. Inspection ensures that only 5% of the chips legally marketed are defective. Unfortunately, some chips are stolen before inspection. If 1% of all chips on the market are stolen, find the probability that a given chip is stolen given that it is defective.

Q3 (Attempt any Two Parts) (5,5)

(a) Let X and Y be jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} 3x + 1, & x, y \geq 0, x + y < 1 \\ 0, & otherwise \end{cases}$$

Find $P(Y < 2X^2)$.

(b) Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} 6xy, & 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x} \\ 0, & otherwise \end{cases}$$

Find the conditional PDF $f_{X|Y}(x|y)$.

(c) Let X denote the number of times a photocopy machine will malfunction on any given month. Let Y denote the number of times a technician is called on an emergency call. The joint probability mass function is presented in the table below:

Y↓ and X→	0	1	2	3
0	0.15	0.30	0.05	0
1	0.05	0.15	0.05	0.05
2	0	0.05	0.10	0.05

(i) Find the probability $P(Y > X)$.

(ii) Are X and Y independent? If not, find Cov(X,Y).