

(Please write your Enrollment Number)

Enrollment No. 08601012019

MID TERM EXAMINATION  
(November, 2019)

Subject Code: BAS 101

Time : 1 ½ Hours

Subject: Applied Mathematics I

Maximum Marks : 30

Note: Q1 is compulsory. Attempt any two questions from the rest.  
Scientific Calculator Allowed

Q1.

- (a) Obtain the Fourier series expansion of  $x \sin x$  as a cosine series in  $(0, \pi)$ . (3,3,2,2)  
(b) Discuss the convergence of the series  $\sum \frac{1}{x^n + x^{-n}}$ ,  $x > 0$ .  
(c) Find the  $n^{\text{th}}$  derivative of  $\frac{1}{x^2 - 6x + 8}$ .  
(d) Prove that Eigen values of a unitary matrix are of magnitude unity.

Q2.

- (a) Find the value of  $n$  so that  $u = r^n(3 \cos^2 \theta - 1)$  satisfies the equation (4,6)  
 $\frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial u}{\partial \theta} \right) = 0$ .  
(b) Solve  $x^2 y z = e$ ,  $x y^2 z^3 = e$ ,  $x^3 y^2 z = e$  using matrices.

Q3.

- (a) Change the differential equation  $x^2 z_{xx} - y^2 z_{yy} = 0$  into one with  $u$  and  $v$  as the independent variables, where  $u = xy$  and  $v = x/y$ . (4,3,3)  
(b) Discuss the convergence of the series  $1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \dots$ , ( $x > 0$ )  
(c) If  $y^{1/m} + y^{-1/m} = 2x$ , prove that  $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$ .

Q4.

- (a) Test the convergence of the series  $\sum_{n=2}^{\infty} \frac{\ln n}{2n^3 - 3}$ . (3,5,2)  
(b) Find the Fourier series of the periodic function  $f(x)$  of period 2, where  $f(x) = \begin{cases} -1 & -1 < x < 0 \\ 2x & 0 < x < 1 \end{cases}$   
and show that  $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ .  
(c) Is the matrix  $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$  diagonalizable?