End-Term Examination

(CBCS)(SUBJECTIVE TYPE)(Offline)

Course Name:<B.Tech. - CSE/ECE> Semester:<1st>

(Feb-March, 2023)

Subject Code: BAS 101	Subject: Applied Mathematics -1
Time :3 Hours	Maximum Marks : 60
Note : Q1 is compulsory. Attempt one question each from the Units I, II, III & IV.	

Q1 (2.5*8=20)

(a) Verify Cayley-Hamilton Theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

(b) Find the eigen values of $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ and using these find the eigen values of A^{-1} and A^3.

(c) Discuss the monotonicity and the convergence of the sequence $\{a_n\}$ where $a_n = \frac{n}{n^2+1}$.

(d) Calculate the approximate value of $\sqrt{26}$ correct to two decimal places using Taylor's series.

(e) If $y = e^x \cos x \cos 2x$, then find y_n.

(f) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

(g) Evaluate $\int_0^1 \int_0^1 \frac{1}{\sqrt{(1-x^2)(1-y^2)}}\, dx\, dy$.

(h) Find the area of the region bounded by the lines $x = 2$, $x = -2$ and the circle $x^2 + y^2 = 9$.

UNIT-I

Q2 (a) Examine the following vectors $[1,2,-2]$, $[-1,3,0]$, $[0,-2,1]$ for linear dependence and find the relation between them, if it exists. (5*2=10)

(b) Reduce the matrix A to its normal form when

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$ Hence find the rank of A.

Q3 (a) For what values of k, the set of equations (5*2=10)

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

have a solution?

(b) Examine the following vectors [1, 2, 4],[2, -1, 3], [0, 1, 2] and [-3,7,2] for linear dependence and find the relation between them, if it exists.

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UNIT-II

Q4 (a) Examine the series **(5*2=10)**

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots$$

for convergence and absolute convergence.

(b) Expand $\frac{e^x}{e^x+1}$ upto third degree term.

Q5 (a) Examine for convergence and absolute convergence of series **(5*2=10)**

$$1 - \frac{1}{2^3}(1+2) + \frac{1}{3^3}(1+2+3) - \frac{1}{4^3}(1+2+3+4) + \cdots$$

(b) If $y = (x^2-1)^n$ then prove that
$(1-x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0.$

UNIT-III

Q6 (a) Trace the curve $y^2(a+x) = x^2(3a-x)$. **(5*2=10)**

(b) Divide a given positive number "a" into three parts (positive) such that their sum is "a" and product is maximum.

Q7 (a) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, show that **(5*2=10)**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

(b) Test the function $f(x,y) = x^3y^2(6-x-y)$ for extreme values.

UNIT-IV

Q8 (a) Evaluate the integral **(5*2=10)**

$$\int_0^a \int_{x^2/a}^{2a-x} xy\, dxdy$$

by changing the order of integration.

(b) Transform the integral

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{(x^2+y^2)}\, dxdy$$

by changing to polar coordinates, and hence evaluate it.

Q9 (a) Evaluate $\int_0^a \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}} dxdy$ by changing the order of integration. **(5*2=10)**

(b) Evaluate $\int_{z=0}^{4}\int_{x=0}^{2\sqrt{z}}\int_{y=0}^{\sqrt{4x-x^2}} dy\, dx\, dz$

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