(Please write your Enrollment Number)

Enrollment No. 0 32011 99023

End-Term Examination

(CBCS)(SUBJECTIVE TYPE)(OffLine)

Course Name: B.Tech , Semester: 3rd Sem.

(December, 2024)

Subject Code: BCS 203	Subject: Discrete Structures
Time :3 Hours	Maximum Marks : 60

Note: Q1 is compulsory. Attempt one question each from the Units I, II, III & IV.

Q1 (5*4=20)

a) Identify the nature of the proposition S, whether it is Tautology/ Contingency/ Contradiction.
S: $((P\wedge Q)\rightarrow R)\rightarrow((P\wedge Q)\rightarrow(Q\rightarrow R))$

b) Draw the Hasse Diagram of the following: D_{105}, and D_{72}

c) Prove that group G is an Abelian group if and only if $(ab)^{-1} = a^{-1}b^{-1}$, $\forall\, a, b \in G$.

d) The chromatic number of the following graph is ______

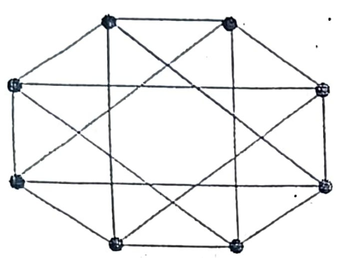


Fig. (a)

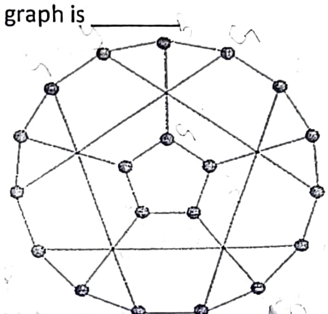


Fig. (b)

UNIT I

Q2 (5+5)

a) Let p, q, r, s represent the following propositions.
p: x∈{8,9,10,11,12}
q: x is a composite number
r: x is a perfect square
s: x is a prime number
The integer x≥2 which satisfies: $\neg((p\Rightarrow q)\wedge(\neg r\vee\neg s))$ is ________.

b) The binary operator ≠ is defined by the following truth table.

p	q	p ≠ q
0	0	0
0	1	1
1	0	1
1	1	0

Identify the nature of the binary operator ≠, whether it is associative, commutative, or both?

Q3 (5+5)

a) Translate the following into propositional logic: i) not all rainy days are cold ii) None of my friends are perfect. Note: Where the variables are: rainy(x), cold(x), f(x): friend, p(x): perfect.

b) Identify the following Boolean expressions which is/are NOT tautology?

A. $((a \to b) \wedge (b \to c)) \to (a \to c)$

B. $(a \leftrightarrow c) \to (\sim b \to (a \wedge c))$

C. $(a \wedge b \wedge c) \to (c \vee a)$

D. $a \to (b \to a)$

UNIT II

Q4 **a) Using the principle of mathematical induction, Show that 2^{2n}- 1 is divisible by 3.** (5+5)

b) Find the total number of relation on a set R with n elements which is antisymmetric but not reflexive.

Q5 ()

a) Prove that the relation congruence modulo m on the set Z of all integers is an equivalence relation.

b) Explain the following Sets with example: a) Finite b) Infinite, c) Countable d) Uncountable.

UNIT III

Q6 (5+5)

a) Define a Field. Prove that the set of integers Z_{11} with addition and multiplication is a Field.

b) State and prove Lagrange's theorem for finite groups.

Q7 (5+5)

a) Prove that a group of prime order p is cyclic.

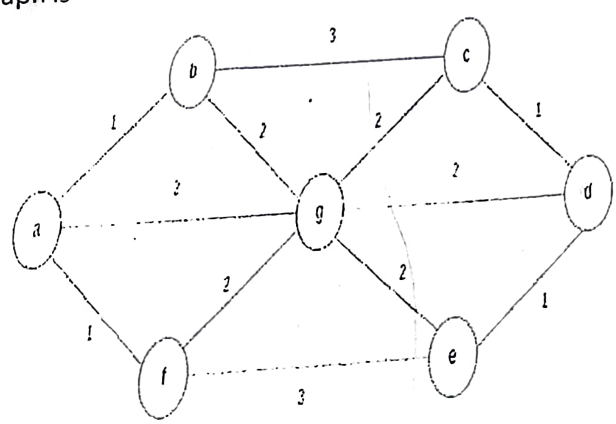
b) Let G be the set of all positive rational numbers and * be the binary operation on G defined as $a * b = \frac{ab}{7}$, $\forall a, b \in G$. Prove that $(G,*)$ be an abelian group.

UNIT IV

Q8 (5+5)

a) State and prove Euler's formula for connected planar graphs: V–E+R= 2, where V, E, and F represent vertices, edges, and region, respectively.

b) The number of distinct minimum-weight spanning trees in the following graph is



Q9 (5+5)

a) G = (V,E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e.

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e.

b) Prove that: Any planar graph can be colored using at most five colors such that no two adjacent vertices share the same color.