END TERM EXAMINATION

FIRST SEMESTER [B.TECH] DECEMBER 2024

Paper Code: BS-111 **Subject: Applied Mathematics-I**

Time: 3 Hours **Maximum Marks: 60**

Note: Attempt any five questions including Q.No1 which is compulsory. Internal Choice is indicated.

Q1. Attempt **any Four** of the following questions: **(4x5=20)**

a) For the transformation $x = a(u+v), y = b(u-v)$ and $u = r^2\cos 2\theta, v = r^2\sin 2\theta$. Find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$

b) Test for consistency the following equation and solve them if consistent $3x+7y+8z=-13, 2x+9z=-5, -4x+y+26z=2$

c) If vector $\vec{v} = (x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal vector, then find the value of a.

d) Find the directional derivative of $f(x,y,z) = xy^3 + yz^3$ at the point (2, -1, 1) in the direction of vector $\hat{i}+2\hat{j}+3\hat{k}$

e) If $z = f(x+ay)+\phi(x-ay)$, then show that $z_{yy} = a^2 z_{xx}$

f) Solve $\dfrac{dy}{dx} = \dfrac{x(x^2+y^2-1)}{y(x^2+y^2+1)}$

g) Define Gamma Function and Prove that $\displaystyle\int_0^\infty e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$

h) Verify the vectors $(1,9,9,8),(2,0,0,3),(2,0,0,8)$ are linearly Independent or not?

Q2 a) If $\dfrac{x^2}{a^2+u}+\dfrac{y^2}{b^2+u}+\dfrac{z^2}{c^2+u}=1$, show that $u_x^2+u_y^2+u_z^2 = 2(xu_x+yu_y+zu_z)$. **(5)**

b) Show that the function $f(x,y) = x^3\exp(-x^2-y^2)$ has a maximum at the point $(\sqrt{3/2},0)$. A minimum at $(-\sqrt{3/2},0)$ and a stationary point at the origin whose nature cannot be determined. **(5)**

OR

Q3 a) Evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}dx$ by applying differentiation under integral sign **(5)**

b) Find the stationary points of $f(x,y,z) = x^3+y^3+z^3$ subject to constraint $g(x,y,z) = x^2+y^2+z^2 = 1$ **(5)**

Q4 a) Solve $\dfrac{dy}{dx}-\dfrac{\tan y}{1+x} = (1+x)e^x\sec y$ **(5)**

b) Solve $\left(x^2y-2xy^2\right)dx=\left(x^3-3x^2y\right)dy$ **(5)**

OR

Q5 a) Solve $\left(D^2+2D+1\right)y=2x+x^2$ **(5)**

b) Apply the method of variation of parameters, to solve $\frac{d^2y}{dx^2}+y=Co\sec x$ **(5)**

Q6 a) Solve the following system by Gauss Elimination method **(5)**

$$2x_1-x_2+3x_3=9$$
$$x_1+x_2+x_3=6$$
$$2x_1-x_2+x_3=2$$

b) If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix}=0$ in which a, b, c are different, show that abc=1. **(5)**

OR

Q7 a) State Cayley-Hamilton Theorem and verify Cayley-Hamilton Theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **(5)**

check

b) Find the diagonalization for the matrix $A=\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ **(5)**

Q8 a) Using Green's theorem find the line integral $\oint_c \left(x^2-2xy\right)dx+\left(x^2y+3\right)dy$ **(5)**

where C is the boundary of the region defined by $y^2=8x$ and $x=2$

b) Find the directional derivative of $f\left(x,y,z\right)=xy^3+yz^2$ at the point (2,-1,1) in the direction of vector $\hat{i}+2\hat{j}+3\hat{k}$ **(5)**

OR

Q9. a) Find the divergence and curl of $F=2x^2z\hat{i}-xy^2z\,\hat{j}+3yz^2\hat{k}$ at the point (1,1,1) **(5)**

b) Find the curvature and torsion for the curve **(5)**
$x=a\sin t,\ y=a\cos t,\ z=at\cot\alpha$

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