(Please write your Exam Roll No.) *Exam Roll No. ....*

END TERM EXAMINATION

FIRST SEMESTER [B.TECH] MARCH 2023

Paper Code: BS-111	***Subject: Applied Mathematics-I***
Time: 3 Hours	***Maximum Marks: 75***

Note: Attempt five questions in all including Q. No.1 which is compulsory. Select one question from each unit. Assume missing data, if any.

Q1 Attempt **all** questions:-

(a) If $\int_0^1 x^m dx = \frac{1}{m+1}$, thenfind the value of $\int_0^1 x^m(\log x)\, dx$. **(2.5)**

(b) If $\vec{F} = (x + y + 1)\hat{\imath} + \hat{\jmath} - (x + y)\hat{k}$, thencompute the value of $\vec{F}.\,\mathrm{Curl}(\vec{F})$. **(2.5)**

(c) Find the particular integral for the linear differential equation: **(2.5)**

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \sin 3x.$$

(d) Determine the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 1 & 2 & 5 & 2 \\ 2 & 1 & 1 & 3 \end{bmatrix}$. **(2.5)**

(e) Applying Gauss divergence theorem, find the value of $\int\int_S \vec{F}.\hat{n}\, ds$, for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - xz)\hat{\jmath} + (z^2 - yx)\hat{k}$ and S is the cube $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$. **(2.5)**

(f) Find the stationary values of the function $f(x, y) = x^3y^2(1 - x - y)$. **(2.5)**

UNIT-I

Q2 (a) If $u = f(r)$ and $x = r\cos(\theta),\ y = r\sin(\theta)$, then prove that **(7)**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$$

(b) Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2}dx$ and hence show that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{\pi}{8}\log 2$. **(8)**

Q3 (a) Find the shortest and longest distances from the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$. **(6)**

(b) If$u = x^2 - y^2,\ v = 2xy$ and $x = r\cos(\theta),\ y = r\sin(\theta)$, then find the Jaccobian$J = \frac{\partial(u,v)}{\partial(r,\theta)}$. **(4)**

(c) If$u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(5)**

UNIT-II

Q4 (a)Solve the ordinary differential equation: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$. **(8)**

(b) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$, where $J_n(x)$ represents the Bessel function of first kind. **(7)**

Q5 (a) Let the electric equipotential lines (curves of constant potentials) between two concentric cylinders be given by $x^2 + y^2 = c$, where c is the constant. Find their orthogonal trajectories (known as curves of electric force). **(4)**

(b) Solve the ODE: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x\log x$, by variation of parameters. **(6)**

(c) Solve the ODE: $2y\,dx + x(2\log x - y)dy = 0$, by choosing suitable method. **(5)**

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UNIT-III

Q6 (a) Test the consistency and solve the system of equations: **(7)**
$3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4.$

(b) Verify Cayley – Hamilton Theorem for the matrix **(8)**
$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, and hence find A^{-1}.

Q7 (a) Check whether the vectors $(2, 1, 1), (2, 0, -1); (4, 2, 1)$ are Linearly dependent or independent? **(6)**

(b) Reduce the quadratic form $2xy + 2yz + 2zx$ into the cannonical form and discuss its nature. **(9)**

UNIT-IV

Q8 (a) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$, in the direction of the normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$. **(7.5)**

(b) Apply Stoke's Theorem to evaluate $\int_C (y\,dx + z\,dy + x\,dz)$, where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. **(7.5)**

Q9 (a) Find the curvature and torsion of the Helix $x = a\cos t,\ y = a\sin t,\ z = bt$. **(7.5)**

(b) Prove that $\mathrm{div}(\mathrm{grad}\, r^n) = \nabla^2(r^n) = n(n+1)r^{n-2}$, where $r = \sqrt{(x^2 + y^2 + z^2)}$. **(7.5)**

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