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DEPARTMENT OF MATHEMATICS
National Institute of Technology, Kurukshetra

Question Paper (Theory)

B.Tech. First Semester
Differential Calculus and Differential Equations
Course Code: MAIC-101

End Semester Examination Dec. 2023
MM: 50
Time: 3 hrs.

Note: All questions are compulsory and carry equal marks.

$\left(x = 2,\ y = 1,\ z = 0\right)$

1. Test for the consistency of the system $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ and solve, if consistent.

$y + 2z = 1$
$2y + 6z = 2$
$y + 3z = 1$
$2z = 0$
$x = 2$
$2 + 1 + \cdots$

2. Find the eigen values and eigen vectors of the following matrix

$(1, -2, -3)$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}.$$

$\begin{bmatrix} 2 & 1 & 9 \\ -3 & 0 & -24 \\ 0 & 0 & 4 \end{bmatrix}$

3. Find the Maxima & Minima of the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

4. Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm.

$432 = xy + 2yz + 2xz$
$\Rightarrow xyz =$
$yz + xz + xy = 0$

5. Solve by method of undetermined coefficients $(D^2 + 1)y = 2\cos x$.

6. Solve by method of variation of parameters $\frac{d^2y}{dx^2} + y = \cosec x$.

7. Write the statement of radioactive decay. Write it mathematically in terms of the differential equation. Also, derive the formula for half-life.

8. A metal ball is heated to a temperature at 100°C and at time t=0 it is placed in water which is maintained at 40°C. If the temperature of the ball is reduced to 60°C in 4 min. Find the time at which temp. of ball is 50°C.

9. Find the inverse Laplace transform of $\frac{2s+1}{(s+2)^2(s-1)^2}$.

10. Solve the given D.E. using Laplace transformation $ty'' + 2y' + ty = \cos t$, given $y(0) = 1$.

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$S = \dfrac{1}{(S-2)^2} - \dfrac{1}{(S-1)^2}$

$V = l \times b \times h$

$(S-1)^2 - (S-2)^2$
$S^2 + 4 - 2S - S^2 - 1 + 4S$
$2S + 3 - 2$

$432 = l \times b + 2(l \times h) + 2(b \times h)$
$2S + 1 + 2$

$s^2 Y(0) + sY(0) + s'Y(0)$