NATIONAL INSTITUTE OF TECHNOLOGY KURUKSHETRA

END-SEMESTER EXAMINATION, MAY/JUNE - 2024

Programme: B.Tech., (ECE, 4th Sem.) Course: **Information Theory and Coding**

Course Code: **ECPC-212** Maximum Marks: **50**

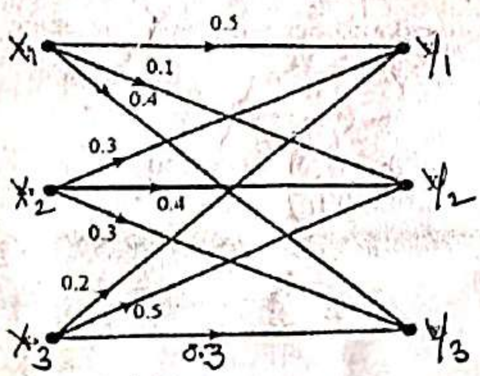
Time Allowed: **3 Hours** Roll No. ____________

Instructions: 1. Attempt *all* questions.
2. All parts of a question should be answered at one place.

1

a) A zero-memory source has symbols, $S=\{s_1, s_2, s_3\}$ with probabilities $\{0.5, 0.3, 0.2\}$, respectively. Find the source entropy. Also, compute the entropy of its second extension. (Note: Don't use the source entropy directly to calculate the entropy of the second order extension of the source)

b) Consider a channel with 3 input symbols X_1, X_2, and X_3 having probabilities $P(X_1)=0.5$ and $P(X_2)=P(X_3)=0.25$, respectively, as shown in figure below. The output symbols are represented as Y_1, Y_2, and Y_3.



I. Find the channel transition matrix
II. Find the output probabilities
III. Find the input entropy $H(X)$
IV. Find the output entropy $H(Y)$
V. Find the conditional entropy $H(Y/X)$

5+5

2

a) Consider a discrete memoryless source of symbols $S=\{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ with probabilities $\{0.45, 0.15, 0.12, 0.08, 0.08, 0.08, 0.04\}$, respectively. Determine the Shannon-Fano and Huffman codes for this source. Also, find the efficiency η of these coding methods.

b) State the channel capacity theorem and derive the channel capacity of a noisy channel (AWGN) under the constraints of transmitted power and bandwidth.

5+5

3

a) Consider the generator matrix

$$G=\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Construct the all eight code words in the dual code. Find the minimum distance of these dual codes. [Note: Here the generate matrix G is represented as $G=[P;I_k]$, where P is the parity matrix and I_k is the identity matrix of size $k \times k$].

b) Consider the (7,4) linear code with parity-check matrix

$$H=\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

Determine the error correcting capability of this code. Also draw the syndrome circuit for this linear systematic code. [Note: Here the matrix H is represented as $H=\left[I_{n-k};P^T\right]$, where P is the parity matrix and I_{n-k} is the identity matrix of size $(n-k)\times(n-k)$]. 5+5

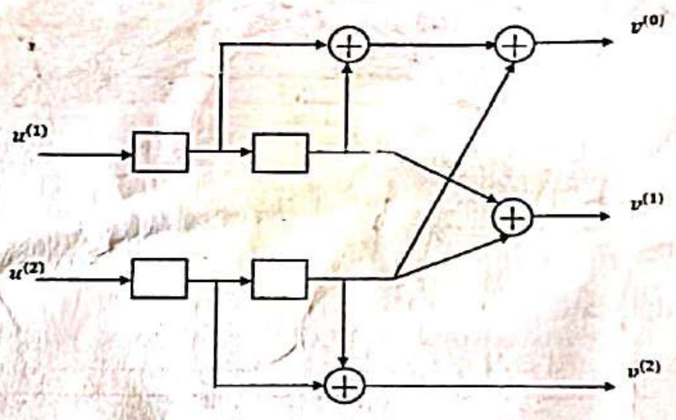
4

a) Consider the (7, 3) cyclic code generated by the polynomial $g(X)=X^4+X^3+X^2+1$. Determine the parity polynomial $h(X)$ of this code. Also, compute the generator matrix and the parity-check matrix for this code.

b) Draw and explain (in detail) the Meggitt decoder for cyclic codes. 5+5

5

a) Consider a convolutional encoder with input bit streams as $u^{(1)}$ and $u^{(2)}$, and the encoder outputs as $v^{(0)}$, $v^{(1)}$, and $v^{(2)}$ as shown in figure below. Each box represents a shift register. Determine the generator polynomial matrix $G(D)$ for this encoder. Evaluate the memory order and the constraint length of the encoder. Also find the outputs corresponding to the inputs $u^{(1)}=(101)$ and $u^{(2)}=(110)$.



b) Draw the convolutional encoder described by the generator polynomial matrix $G(D)=\left[1+D+D^2 \quad 1+D^2\right]$. 7+3