II Sem.- B.Tech.

MID-SEMESTER EXAMINATION, May 2023

Course Code- CACSC01/CMCSC02/COCSC01/CDCSC01

Time- 1.5 Hours

Course Title- Discrete Structures **Max. Marks- 25**

Note: - Attempt all the five questions. Missing data/ information if any, maybe suitably assumed & mentioned in the answer.

Q.No	Attempt all	Marks	CO
1(a)	Show that the argument form with premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$ and conclusion $q \rightarrow r$ is valid.	2.5	1
1(b)	Let S(x) be the predicate "x is a student," F(x) the predicate "x is a faculty member," and A(x, y) the predicate "x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express each of these statements. a) Some student has not asked any faculty member a question. b) There is a faculty member who has never been asked a question by a student. c) Some student has asked every faculty member a question. d) There is a faculty member who has asked every other faculty member a question. e) Some student has never been asked a question by a faculty member.	2.5	1
2(a)	State whether $\exists x \forall y\, P(x, y)$ is equivalent to $\forall y \exists x\, P(x, y)$, where P (x, y) is a predicate. Justify with suitable example.	2.5	1
2(b)	Validate the statements: Each living thing is a plant or animal. David's dog is alive and it is not a plant. All animals have heart. Hence, David's dog has a heart.	2.5	1
3(a)	Define partition of a set and show that an equivalence relation on a set partitions the given set. Set of subset (non empty) s.t. $a_1 \cup a_2 = a$, $a_1 \cap a_2 = \phi$	2.5	1
3(b)	Let $R=\{(x, y) \mid x, y \in Z$ and $x - y$ is divisible by $n\}$. Show that R is an equivalence relation. Find a partition of Z.	2.5	1
4(a)	What is pigeonhole principle? Prove that if 101 integers are selected from the set A = {1,2,...,200}, then there are two integers such that one divides the other.	2.5	2
4(b)	Find the number of non-negative integer solutions to the equation: $x_1 + x_2 + x_3 + x_4 = 13$ with the extra condition that $x_i \leq 5$, for all $1 \leq i \leq 4$.	2.5	2
5(a)	Using generating function, solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.	2.5	2
5(b)	A person deposits Rs 1000 in an account that yields an interest of 9% compounded annually. Set up a recurrence relation for the amount in the account at the end of n years. Find an explicit formula for the amount in the account at the end of n years. How much money will the account contain after 100 years?	2.5	2

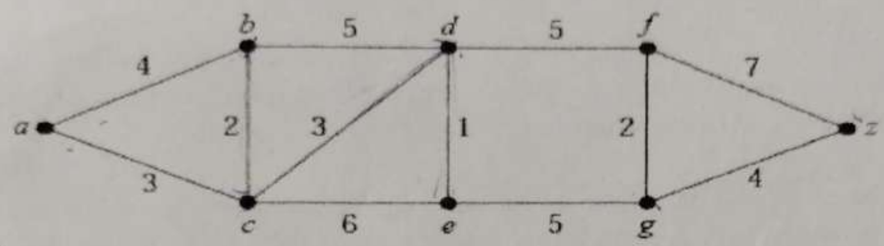
Second Semester-B.Tech Course
End Semester Examination, July, 2023

Course code: CACSC01/CDCSC01/COCSC01/CMCSC03
Course title: Discrete Structures
Time: 3 hours. Maximum Marks. 50

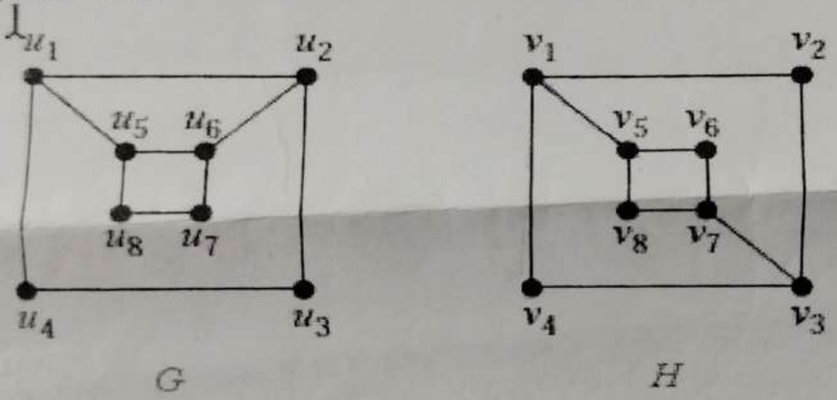
Note: Missing data/information(if any), may be suitably assumed and mentioned in the answer.

Q. No.	Question	Marks	CO
Q1	Attempt any 2 parts of the following:		
1a	Validate the argument: If camels fly or goat eat grass, then the mosquito is the national bird. If the mosquito is the national bird, then peanut butter tastes good on burgers. But, peanut butter tastes terrible on burgers. Therefore, the goat does not eat grass.	5	CO1
1b	(I) Suppose that the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is the 1, 2 or 3, and y is 0, 1, 2 or 3. Write out each of these propositions using disjunctions, conjunctions, and negations. a) $\exists\, x\, P(x, 2)$ b) $\forall\, y\, P(1, y)$ c) $\exists\, y \sim P(4, y)$ d) $\forall\, x \sim P(x, 3)$. (II) Prove that $R \vee S$ is tautology implied by the premises $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ using rules of inference.	3+2	CO1
1c	Find $\cup_{i=1}^{\infty} A_i$ and $\cap_{i=1}^{\infty} A_i$ if for every positive integer i, where $A_i = \{-i, -i+1, ..., -1, 0, 1, ..., i-1, i\}$. Prove or disprove that if x is a real number, then x^2 is a positive real number.	3+ 2	CO1
Q2	Attempt any 2 parts of the following:		
2a	Define generating function. Using this, find an explicit formula for a_n where $\{a_n\}$ satisfies the recurrence relation $a_n = 8a_{n-1} + 10^{n-1}$ and the initial condition $a_0 = 1$.	5	CO2
2b	Use Mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every non-negative integer n.	5	CO2
2c	Define Big- O notation. Show that for $x \in \mathbb{R}$, $7x^2 + 5x + 3$ is $O(x^3)$. Is it true that x^3 is $O(7x^2)$?	5	CO2
Q3	Attempt any 2 parts of the following:		
3a	Let D_{60} be the set of all positive divisors of 60. Then answer the following: (i) Draw the Hasse diagram of D_{60}. (ii) Is D_{60} a distributive lattice? (iii) Find complement of each element of D_{60}, if exist. (iv) Is D_{60} a Boolean algebra?	5	CO3

3b	Prove the Boolean identities: $ABC + AB\overline{C} + A\overline{B}C + \overline{A}BC = AB + BC + CA$	5	CO3
3c	State Division algorithm. Hence, find the gcd of 1529 and 14039, using Euclidean algorithm. Also, Find the Bezout's coefficients.	5	CO3
Q4	Attempt any 2 parts of the following:		
4a	State Handshaking Theorem. Show that the number of edges in a complete graph with n vertices is $\frac{n(n-1)}{2}$.	5	CO4
4b	Use Dijkstra's algorithm to find the shortest path between a and z in the given weighted graph.	5	CO4



4c	Define graph isomorphism. Check whether the graphs G and H are isomorphic or not. Justify your answer.	5	CO4



Q5	Attempt any 2 parts of the following:		
5a	A random variable X has the following probability function:	5	CO5

x	0	1	2	3	4	5	6	7
p(x)	0	k	$2k$	$3k$	$3k$	k^2	$2k^2$	$7k^2 + k$

i) Find k,

ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$.

5b	In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p (probability of success) of the distribution. Also, evaluate its mean and variance.	5	CO5
5c	A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which i) neither car is used, and ii) the proportion of days on which some demand is refused.	5	CO5