Roll No. 2400100 7034 .

Total Pages : 04

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December 2024

B.Tech. (Electrical) (First Semester)

Mathematics-I (Calculus and Differential Equations) (BSC103C)

Time : 3 Hours] [*Maximum Marks : 75*

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) Evaluate the integral $\int_0^1 x^5(1-x^3)^{10}\,dx$. **1.5**

(b) Use Taylor's Theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. **1.5**

(c) Write a short note about Cauchy's integral test and log test of infinite series. 1.5

(d) Prove that $div\left(\frac{\vec{r}}{r^3}\right)=0$. 1.5

(e) Evaluate $\iint e^{2x+3y}\,dx\,dy$ over the triangle $x = 0$, $y = 0$ and $x + y = 1$. 1.5

(f) State Stoke's Theorem. 1.5

(g) Solve the differential equation : 1.5

$$(2xy+e^y)dx+(x^2+xe^y)dy=0.$$

(h) Define Clairaut's equation. Also find the general solution of the equation $p=\log(px-y)$. 1.5

(i) Evaluate the integral : 1.5

$$\int_0^{\pi/2}\int_0^{a\cos\theta} r\sin\theta\,dr\,d\theta$$

(j) Solve by the method of separation of variables $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}=0$. 1.5

Part B

2. (a) Show that the equation of the evolute of the parabola $x^2 = 4ay$ is $4(y - 2a)^3 = 27ax^2$. 8

(b) Verify Rolle's Theorem for the function $f(x) = x^3 - 3x^2 - x - 3$ in [1, 3] and also find the appropriate value of c. 7

3. (a) Discuss the convergence of the series : 8

$$\frac{a+x}{1!}+\frac{(a+2x)^2}{2!}+\frac{(a+2x)^3}{3!}+\ldots\ldots\infty$$

(b) Find the Fourier series expansion of $f(x) = x + x^2$ for $-\pi < x < \pi$. Deduce that : 8

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots$$

4. (a) Prove that of all the rectangular parallelepipeds of the given volume, the cube has the least surface. 8

(b) Using Taylor's series, expand $\sin x$ in powers of $\left(x-\frac{\pi}{2}\right)$. Hence find the value of $\sin 91°$ correct to 4 decimal places. 7

5. (a) Find the volume of the ellipsoid : 8

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

(b) Verify Divergence Theorem for $\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$, taken over the rectangular parallelepiped by $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. 7

6. (a) Find the power series solution of $(1 - x^2)y'' - 2xy' + 2y = 0$ about $x = 0$. 8

(b) Solve the differential equation $(mz - ny)p + (nx - lz)q = ly - mx$. 7

7. (a) Solve $(p^2 + q^2)y = qz$. 8

(b) Prove that $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer. 7