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December, 2019
B.Tech. (ME/MA/AE) 1st Semester
MATHEMATICS: Calculus and Linear Algebra
(BSC103A)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.
2. Answer any four questions from Part-B in detail.
3. Different sub-parts of a question are to be attempted adjacent to each other.

PART - A

1. (a) Define evolutes and involutes with example. (1.5)

    (b) Evaluate $\int_{0}^{\pi/2} \sqrt{\tan \theta}\, d\theta$. (1.5)

    (c) State Lagrange's Mean Value theorem. (1.5)

    (d) Evaluate $\lim_{x \to \infty} \frac{x^n}{e^x}$. (1.5)

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(e) If $<a_n>$ is bounded and $b_n \to 0$, then $a_n b_n \to 0$.

(1.5)

(f) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n . x^{2n}$. (1.5)

(g) Show that $\lim_{(x,y) \to (o,o)} \left(\frac{x^2}{x^4 + y^2}\right)$ does not exist. (1.5)

(h) Show that the vector

$$\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$$

is irrotational. (1.5)

(i) If 2 and 3 are eigen values of

$$A = \begin{array}{|c|c|c|}
\hline
3 & 10 & 5 \\
\hline
-2 & -3 & -4 \\
\hline
3 & 5 & 7 \\
\hline
\end{array},$$

find the eigen values of A^{-1} and A^3. (1.5)

(j) Find rank of matrix

1	−7	3	−3
7	20	−2	25
5	−2	4	7

by using determinant. (1.5)

PART - B

2. (a) Find the evolute of the rectangular hyperbola $xy = c^2$.

(8)

(b) A sphere of radius a is divided into two parts by a plane at a distance $\frac{a}{2}$ from the centre. Show that the ratio of the volume of two parts is $5 : 27$. (7)

3. (a) Expand $\sin x$ as a finite series in powers of x, with remainder in Lagrange's form. Hence, find the series for $\sin x$. (7)

(b) Using Rolle's theorem, prove that there is no real a for which the equation $x^2 - 3x + a$ has two different roots in $[-1, 1]$. (8)

4. (a) Test the convergence of the series given below :

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-1} + \ldots\ldots\infty \qquad (8)$$

(b) Find the half-range cosine series for $f(x)=x$ in the interval $[0, \pi]$ and deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}. \qquad (7)$$

5. (a) Test the continuity of the function $f(x,y)=\dfrac{xy}{\sqrt{x^2+y^2}}$, if $(x, y) \neq (0,0)$ and $f(0,0)=0$ at the origin. (10)

(b) Discuss the maxima and minima of

$$f(x,y)=x^3y^2(1-x-y). \qquad (5)$$

6. (a) For what value of k, the equations $x+y+z=1$, $2x+y+4z=k$ and $4x+y+10z=k^2$ have (i) unique solution, (ii) infinite number of solutions, (iii) no solution and solve them completely in each case of consistency. (10)

(b) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

Hence, find the matrix whose eigen values are $\dfrac{1}{6}$ and -1. (5)

7. (a) Evaluate $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2} dx$, if it exists. (7)

(b) Find the radius of convergence of the series

$\displaystyle\sum_{n=0}^{\infty} \left(\dfrac{(-1)^n}{8^n} x^{3n} \right)$ and the interval of convergence. (8)

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