Roll No. 21001020004

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B.Tech. (RAI/ME)-I SEMESTER

Mathematics-I (Calculus and Linear Algebra) (BSC-103A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Describe rank of a matrix A with numerical example. (1.5)

   (b) State Rolle's Theorem. (1.5)

   (c) Expand the function log x using Taylor series. (1.5)

   (d) What is relation between Beta and Gamma function. (1.5)

   (e) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n. \qquad (1.5)$$

(f) Explain Fourier series of a function. (1.5)

(g) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$ (1.5)

(h) Find the divergence of the vector $\vec{V} = xyz$. (1.5)

(i) Explain Eigenvalues and Eigenvectors of square matrix A. (1.5)

(j) What are the Eigenvalues of the Hermitian matrix. (1.5)

PART-B

2. (a) For what values of k, the equations

$$x + y + z = 1, \quad 2x + y + 4z = k$$

and $4x + y + 10z = k^2$ have

(i) a unique solution,

(ii) infinite number of solutions,

(iii) no solution,

and solve them completely in each case of consistency. (7)

(b) If $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$,

then find the Eigen values of $A^2 - 2A + I$. (8)

3. (a) Find the extreme values of the function

$$f(x, y) = x^3 + y^3 - 12x - 3y + 20.$$ (7)

(b) Find a unit normal to the surface $xy^3z^2 = 4$, at the point $(-1, -1, 2)$. (8)

4. (a) Find the Fourier series for the function $f(x) = x^2$, $-\pi < x < \pi$. Hence, show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}.$$ (7)

(b) Test the convergence of the following series

(i) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$.

(ii) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$. (8)

5. (a) What will be the value of c of Lagrange's mean value theorem for the function $f(x) = x^3 + x$ in $[1, 2]$. (7)

(b) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sec x)^{(\cot x)}$. (8)

6. (a) Will the improper integral $\int_1^{\infty} \frac{\log x}{x^2}$ be convergent or not? (7)

(b) (i) Find the value of $\int_0^1 x^7 (1-x)^6 \, dx$.

(ii) What will be the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ along the major axis. (8)

7. (a) Let $f: R^2 \to R$ be defined by setting

$$f(x,y)=\frac{xy}{\sqrt{(x^2+y^2)}},$$

when $(x, y) \neq (0, 0)$, $f(0, 0) = 0$

Show that f_x and f_y exist at (0, 0), also, check that the continuity of the function f at origin. (7)

(b) Find the equation of the evolute of the parabola $y^2 = 4ax$. (8)
