JCBUST, YMCA (Department of Mathematics)
First Sessional Test – Oct 2023 (Semester: First)
Subject: Mathematics I (BSC-103 A & BSC-103 RAI)
B. Tech. (Mechanical & Robotics and Artificial Intelligence)

Time: 90 Minutes

Max. Marks: 15

**Note: Attempt All Questions:**

1. For what values of k, the equations

$$x+y+z=1,\quad 2x+y+4z=k,\ \text{and}$$
$$4x+y+10z=k^2$$

have

(a) Unique solution

(b) Infinite number of solutions

(c) No solution

and solve them completely. [5] [CO5]

2. If 2 and 3 are eigenvalues of $A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$, find the Eigen values of $A^{-1}$, adjoint of $A$ and $A^3+2A+I$. [5] [CO5]

3. If $a_0, a_1, a_2, \ldots, a_n$ are real numbers such that

$$\frac{a_0}{n+1}+\frac{a_1}{n}+\frac{a_2}{n-1}+\ldots+\frac{a_{n-1}}{2}+a_n=0,$$

then there exists at least one $x$ in $(0,1)$ such that

$$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n=0.$$ [3] [CO2]

4. Evaluate $\lim_{x\to 0}\log_{\tan 2x}\tan 3x$. [2] [CO1]