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B.Tech. (ECE/EIC/ECC/FAE) 1st SEMESTER

Mathematics-I (Calculus and Linear Algebra)

(BSC-103D)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Evaluate $\int x^2 \sin 2x dx$.

(b) Using property of Beta function, Evaluate

$$\int_0^1 x^{11}(1-x)^5 dx.$$

(c) Find the maximum and minimum values of $x^5 - 5x^4 + 5x^3 - 1$.

(d) Find the Taylor's series expansion of $\sin x$ about $x = \pi/4$.

(e) Find the half range cosine series for

$$f(x) = x,\ 0 \leq x \leq \pi.$$

(f) Prove that the exponential series

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots\dots + \frac{x^n}{n!} + \dots\dots\infty.$$

(g) Show that the vector

$$\vec{f} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$$

is irrotational.

(h) If $\phi = x^2y + xy^2 + z^2$, then find grad ϕ at (1, 1, 1).

(i) Find the inverse of $A = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -2 & 4 \\ 3 & 1 & 2 \end{bmatrix}$.

(j) Find the Eigen value of the given matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

(1.5×10=15)

PART-B

2. (a) Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid. (8)

(b) The area bounded by $y^2 = 4x$ and the line $x = 4$ is revolved about the line $x = 4$. Find the volume of the solid of revolution. (7)

3. (a) Using L'Hospital rule, solve the indeterminant form $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$. (8)

(b) Using Rolle's theorem, prove that there is no real 'a' for which the equation $x^2 - 3x + a = 0$ has two different roots in $[-1, 1]$. (7)

4. (a) Test the convergence and divergence of the series :

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots\ldots\infty, \quad x > 0. \qquad (8)$$

(b) Find the radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$. (7)

5. (a) Find the shortest and longest distance from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$ using Lagrange's method of undetermined multipliers. (8)

(b) If $r^2 = x^2 + y^2 + z^2$, then prove that

$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}.$$ (7)

6. (a) For what values of k, the equations $x + y + z = 1$, $2x + y + 4z = k$ and $4x + y + 10z = k^2$ have

(i) Unique solution

(ii) infinite number of solutions

(iii) no solution. (8)

(b) Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$ Also find A^{-1}. (7)

7. (a) Diagonalize the given matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$ (8)

(b) Find the equation of the tangent plane and the normal to the surface of $z^2 = 4(1 + x^2 + y^2)$ at (2, 2, 6). (7)
