22001008023

Sr. No. 008102

March 2023

B.Tech. 1st SEMESTER

Mathematics-I(Calculus and Linear Algebra)(BSC-103D)

Time: 3 Hours

Max. Marks:75

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

Que.1(a)Evaluate the definite integral $\int_0^2 e^x dx$ as the limit of a sum.

(b)State the relation between Beta and Gamma function.

(c)Using L'Hospital rule , evaluate $\lim_{x\to 0} x\log x$.

(d)Find the maximum and minimum value of the function $f(x) = \sin 2x+5$.

(e)Define Absolute and conditional convergence of an infinite series by giving one example each.

(f)Define Power series and radius of convergence of power series.

(g)Find gradient of Φ at the point (1, 1, 1), where $\Phi(x,y,z) = x^2y+y^2x+z^2$.

(h)Define irrotational and solenoidal vectors.

(i)State rank nullity theorem.

(j)Define symmetric and skew symmetric matrices. Also give one example of each.

(1.5*10=15)

PART-B

Que.2(a)For the given rectangular hyperbola $xy = a^2$,

(i)Find the radius of curvature (ρ).

(ii)Find the coordinates of the centre of curvature (i.e. $\overline{x}, \overline{y}$).

(iii)Show that the evolute of the given curve is $(x+y)^{2/3} - (x-y)^{2/3} = (4a)^{2/3}$. **(10)**

(b)Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the major axis.

(5)

Que.3(a)Verify the Rolle's theorem for $f(x) = \cos 2x$ in $(-\pi/4, \pi/4)$. **(7)**

(b)Verify the Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in (0, 1). **(8)**

Que.4(a)Discuss the convergence of the given infinite series: (7)

$$1+\frac{2^2}{3^2}+\frac{2^2.4^2}{3^2.5^2}+\frac{2^2.4^2.6^2}{3^2.5^2.7^2}+\ldots\ldots\ldots\ldots$$ (8)

(b)Find the Fourier series of the function $f(x) = x - x^2$, $-1 < x < 1$.

Que.5(a)If $u = f(r)$, where $r^2 = x^2+y^2$, then show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = f''(r)+\frac{1}{r}f'(r)$.(7)

(b) Using method of Lagrange's multiplier , find the maximum and minimum value of $u = x^2+y^2+z^2$ subject to the condition $xy + yz + zx = 3a^2$. (8)

Que.6(a)Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -4 \\ -2 & 3 & 7 & -1 \\ 1 & 9 & 16 & -13 \end{bmatrix}$. (7)

(b)Verify the Cayley-Hamilton theorem for the given matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.Also find A^{-1}. (8)

Que.7(a)Using Taylor's series ,expand sinx in powers of $(x- \pi/2)$.Hence find the value of $\sin 91^\circ$ correct to four decimal places. (7)

(b)Diagonalize the given matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$. (8)