Roll No. .... Total Pages : 05

008102

Dec. 2025

B. Tech. (ENC/ECE/EE(IoT) (First Semester)

MATHEMATICS (Calculus And Linear Algebra)

(MTU-152-V/BSC-103D)

Time : 3 *Hours*] [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) Evaluate : **1.5**

$$\int_0^1 x^4\left(1-x^2\right)^{3/2} dx$$

(b) Solve : **1.5**

$$\int_0^1 \frac{\sin^{-1} x}{x} dx$$

(c) Verify Rolle's theorem for $f(x)=\sin x/e^x$ in $(0,\pi)$. **1.5**

$\sin\theta = x$

$\sin x = t$ $\cos\theta \cdot \frac{d\theta}{dx} = 1$ $\frac{t}{\sin t}$

$\sin t$ $\cos\theta \cdot dx$

(d) Find the value of : **1.5**

$$\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$$

(e) Using Cauchy's root test, test the convergence of the given series : **1.5**

$$\sum \frac{n^3}{3^n}$$

(f) Define power series and interval of convergence. **1.5**

(g) Find the first and second partial derivative of $z = x^3 + y^3 - 3axy$. **1.5**

(h) Find the directional derivative of $f(x,y,z) = xy^3 + yz^3$ at the point (2, – 1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **1.5**

(i) Find the rank of the given matrix : **1.5**

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$$

5 20
0 -20 28
3 12 15
-10 -20
0 -10 14
7-12 22-15
-10+10 14+10 2
2 8 10 -5 7
0 -2, 22 6 8

(j) Using Gauss-Jordon method, find the inverse of the given matrix : **1.5**

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

Part B

2\. (a) State and Prove that relation betwen Beta and Gamma functions. **7**

(b) Find the volume formed by the revolution of loop of the curve $y^2(a+x) = x^2(3a-x)$, about the x-axis. **8**

3\. (a) Define Taylor's theorem. Also expand log sin x in powers of $(x-2)$. **7**

(b) Examine the polynomial function $10x^6 - 24x^5 + 15x^4 - 40x^3 + 108$ for maximum and minimum values. **8**

4\. (a) Test the convergence of the given series : **7**

$$\sum \frac{4.7.......(3n+1)}{1.2.........n} x^n.$$

(b) Obtain the half-range sine series for $f(x) = x \cos x$ in $(0, \pi)$. **8**

5. (a) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere : 7

$$x^2 + y^2 + z^2 = 1$$

(b) Calculate (i) curl (grad f), given : 8

$$f(x, y, z) = x^2 + y^2 - z$$

(ii) curl (curl A) given :

$$A = x^2 y\hat{i} + y^2 z\hat{j} + z^2 y\hat{k}$$

6. (a) Define Evolute. Also find the coordinates of the centre of curvature at any point of the parabola $y^2 = 4ax$. Hence show that its evolute is $27ay^2 = 4(x - 2a)^3$. 7

(b) Verify the Cayley-Hamilton theorem for the given matrix and also find its inverse : 8

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

7. (a) Test for the consistency and solve the given system of linear equation : 7

$$5x + 3y + 7z = 4$$

$$3x + 26y + 2z = 9$$

$$7x + 2y + 10z = 5$$

(b) Find the matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form.

Hence calculate A^4. 8

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4 6 8
4 3 1
4-4 3-6 1-8
0 -3 -7