Roll No. ....   Total Pages : 4

80012

Dec., 2018

B.Tech. Ist Semester

MATHEMATICS–I

(HAS-103-C)

Time : 3 Hours]   [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Find the rank of $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$. (1.5)

(b) If $u = x^2 + 1$, $v = y^2 - 2$, then find $\frac{\partial(u, v)}{\partial(x, y)}$. (1.5)

(c) Examine whether $v(x,y) = \dfrac{x^2 - y^2}{2xy}$, is homogeneous.

(1.5)

(d) Evaluate $\int_{-1}^{1} \int_{0}^{1-x^2} (3x^2 + 2y)dy\,dx$. (1.5)

(e) Show that the vector field $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} = 3xz^2\hat{k}$ isirrotational. (1.5)

(f) Evaluate $\int_{0}^{1} \int_{\sqrt{y}}^{1} \cos x^3 \, dxdy$. (1.5)

(g) Show that the radius of curvature at the point $a/4$, $a/4$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $\dfrac{a}{\sqrt{2}}$. (1.5)

(h) Expand $(x - 1)e^x$ near $x = 1$ up to two terms. (1.5)

(i) Find grad Φ when $\Phi = \log(x^2 + y^2 + z^2)$ at (1,2,1). (1.5)

(j) Find the eigen values of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (1.5)

PART–B

2. (a) Verify Stokes theorem for the function $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\,\hat{j}$ in the region bounded by the lines $x = 0$, $y = 0$, $x = a$, $y = b$. (7)

   (b) Compute grad f and verify that curl(grad f) = 0 where $f(x,y,z) = 16xy^3z^2$. (8)

3. (a) Expand $f(x, y) = \tan^{-1}(y/x)$ in the powers of $(x - 1)$ and $(y - 1)$ upto third degree terms. Hence compute $f(1.1, 0.9)$, approximately. (7)

   (b) Differentiating $\int_0^x \frac{dx}{(x^2 + a^2)^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ under the integral sign. Evaluate $\int_0^x \frac{1}{(x^2 + a^2)^2}\,dx$. (8)

4. (a) Solve by rank method the system of equations $x + 2y - 5z = -9$, $3x - y + 2z = 5$, $2x + 3y - z = 3$, $4x - 5y + z = -3$.

   (b) Find the characteristic equation of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and show that A satisfies the equation. Hence evaluate A^{-1} and A^4. (7)

5. (a) Find the Asymptotes of $4(x^4 + y^4) - 17x^2 y^2 - 4x(4y^2 - x^2) + 2(x^2 - 2) = 0$. (7)

(b) Show that the radius of curvature ρ at P on an ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by $\rho = \frac{CD^3}{ab}$ where CD is the

semi -diameter conjugate to CP. (8)

6. (a) Evaluate $\iint \sqrt{a^2 - x^2 - y^2}\, dxdy$ over the semi-circle

$x^2 + y^2 = ax$ in the positive quadrant. (7)

(b) Prove that $\Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n}} \cdot \frac{\Gamma(2n+1)}{\Gamma(n+1)}$. (8)

7. (a) Verify Euler's Theorem for the functions $f(x, y) =$

$$\frac{x^2(x^2 - y^2)^3}{(x^2 + y^2)^3}.$$

(b) State Cayley- Hamilton theorem and using the theorem find the inverse of

$$A = \begin{bmatrix} 13 & -3 & 5 \\ 0 & 4 & 0 \\ -15 & 9 & -7 \end{bmatrix}$$
