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December, 2019
B.Tech. 1st SEMESTER (Reappear)
Mathematics-I (HAS-103C)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Use Gauss-Jordon method to find the inverse of the given matrix:

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$

(b) Prove that the eigen values of an idempotent matrix are either zero or unity.

(c) Show that the matrices A = $\begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$ are similar.

(d) Expand log sin $(x + h)$ using Taylor's series.

(e) If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

(f) If $V = \frac{x^3y^3}{x^3+y^3}$ then show that $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = 3V$.

(g) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$.

(h) Evaluate $\int_0^a \int_0^a \int_0^a (yz+zx+xy)dxdydz$.

(i) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4).

(j) Find the divergence and curl of the vector $\vec{V} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at the point (2, –1, 1). (1.5×10=15)

PART-B

2\. (a) Test the consistency of the equation $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$. (8)

(b) Diagonalise the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ and obtain the modal matrix. (7)

3\. (a) (i) Find the radius of curvature at any point of the curve $r^n = a^n \cos n\theta$. (8)

(ii) Find all the asymptotes of the given curve:

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0.$$

(b) If $V = f(r)$ and $r^2 = x^2 + y^2 + z^2$, then prove that $V_{xx} + V_{yy} + V_{zz} = f''(r) + \frac{2}{r}f'(r)$. (7)

4\. (a) By changing the order of integration, evaluate

$$\int_0^a \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}}\,dxdy. \quad (8)$$

(b) Using Beta and Gamma function, Prove

(i) $\int_0^{\pi/2} \sin^3 x \cos^{5/2} x dx = \frac{8}{77}$.

(ii) $\int_0^1 x^3(1-x)^{4/3} dx = \frac{243}{7280}$. (7)

5. (a) Compute the line integral $\int_c (y^2 dx - x^2 dy)$ about the triangle whose vertices are (1, 0), (0, 1) and (–1, 0). (8)

(b) Verify the divergence theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$. (7)

6. (a) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 1 & 1 \end{bmatrix}.$$ Show that the equation satisfied by A. Also find A^{-1}. (8)

(b) Prove that if the perimeter of a triangle is constant, then its area is maximum when the triangle is equilateral. (7)

7. (a) Find the smaller of the areas bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$. (8)

(b) Verify the Stoke's theorem for the vector field integrated round the rectangle in the plane $z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = a$, $y = b$. (7)