Roll No. .... Total Pages : 4

008107

April 2022

B.Tech-1st Semester (Reappear)

Mathematics-I (HAS-103C)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Find the rank of the matrix. $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

(b) Find all the eigen values of the given matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(c) Using Taylor's series expansion, expand $\tan^{-1}x$ in powers of $(x-1)$ upto four terms.

(d) Find the radius of curvature at the origin for

$$x^3 + 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0.$$

(e) If $u = x^2y + y^2z + z^2x$, then prove that

$$u_x + u_y + u_z = (x + y + z)^2$$

(f) If $u = x + y + z$, $uv = y + z$, $uvw = z$, then using Jacobian show that $\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.

(g) Evaluate $\displaystyle\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2 \sin\theta\, dr\, d\theta$

(h) Evaluate $\displaystyle\int_0^1 x^4(1-\sqrt{x})^5\, dx$ using Beta Gamma function.

(i) Show that the vector field $\vec{F} = \dfrac{\vec{r}}{r^3}$ is irrotational.

(j) Find curl F where F = grad$(x^3 + y^3 + z^3 - 3xyz)$.

(1.5×10=15)

PART-B

2. (a) Show that the given system is consistent and also solve them :

$$x + y + 4z = 6,$$
$$3x - 2y - 2z = 9,$$
$$5x + y + 2z = 13. \quad (7)$$

(b) Diagonalise the matrix

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix} \qquad (8)$$

3. (a) Find all the asymptotes of the curve

$$x^3 - y^2 - 2xy + 2x - y = 0. \qquad (7)$$

(b) Find the center of curvature of the parabola $y^2 = 4ax$ at the point (x, y). (8)

4. (a) If $r^2 = x^2 + y^2 + z^2$, then prove that

$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}. \qquad (7)$$

(b) A rectangular box open at the top have 32 c.c capacity. Find the dimensions of the box requiring least material for its construction. (8)

5. (a) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same. (7)

(b) Evaluate $I = \int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$ by using Triple integration. (8)

6. (a) A vector field given by $\vec{f} = \sin y\,\hat{i} + (1 + \cos x)\,\hat{j}$, evaluate the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$. (7)

(b) Evaluate by Green's theorem

$$\oint_c [(3x - y)dx + (2x + y)dy],$$

where c is the curve $x^2 + y^2 = a^2$. (8)

7. (a) Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. (7)

(b) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. (8)