Sr. No- 001104

December 2023

B.Tech(AE/CIVIL/CSE) (Re-Appear) 1st SEMESTER

Mathematics-I(HAS-103C)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

Q.1(a)Find the characteristic equation for the given matrix, $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(b)Define similar matrices.

(c)Find the rank of the given matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

(d)Using Taylor's series expand, e^x in powers of (x-2).

(e)Find the radius of curvature at the origin for the given curve:

$2x^4 + 4x^3y + xy^2 + 6y^3 - 3x^2 - 2xy + y^2 - 4x = 0$.

(f)If $u = x^2 - 2y$, $v = x + y$, then prove that Jacobian i.e. $\frac{\partial(u,v)}{\partial(x,y)} = 2x + 2$.

(g)Evaluate the double integral, $\int_0^3 \int_1^2 xy(1 + x + y)dydx$.

(h)Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

(i)Find gradΦ, when $\Phi = 3x^2y - y^3z^2$ at the point (1,-2,-1).

(j) If the vector $\vec{F} = (ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal, find the value of 'a'. (1.5*10 =15)

PART-B

Q.2(a) Check the consistency of the given system of equations: (7)

$2x + 6y = -11$, $6x + 20y - 6z = -3$, $6y - 18z = -1$.

(b) Show that the matrix, $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is diagonalizable. Also obtain the modal matrix. (8)

Q.3(a)If $u = tan^{-1}\left(\frac{y^2}{x}\right)$, *then show that* $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -sin2u sin^2 u$. (7)

(b)Evaluate the integral $\int_0^1 \frac{x^{\alpha} - 1}{log x}dx$ by applying differentiation under the integral sign. $(\alpha \geq 0)$. (8)

Q.4(a)Change the order of integration in the given integral, $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dydx$ and also evaluate after change the integral. (7)

(b)Find by triple integration, the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$. (8)

Q.5(a)Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ, where Q is the point (5,0,4). (7)

(b)If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, then evaluate $\iiint_V \nabla.\vec{F}dV$,where 'V' is bounded by the planes x = 0,y = 0,z = 0 and 2x + 2y + z = 4. (8)

Q.6(a)Using Cayley-Hamilton theorem, find A^4 if $A = \begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$. (7)

(b)Given x+y+z = a, then find the maximum value of $x^m y^n z^p$(use Lagrange's method of multipliers). (8)

Q7(a)State and Prove the relation between Beta and Gamma function. (7)

(b)Verify the Green's theorem in the plane for $\oint_C (2xy - x^2)dx + (x^2 + y^2)dy$, where c is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$. (8)