

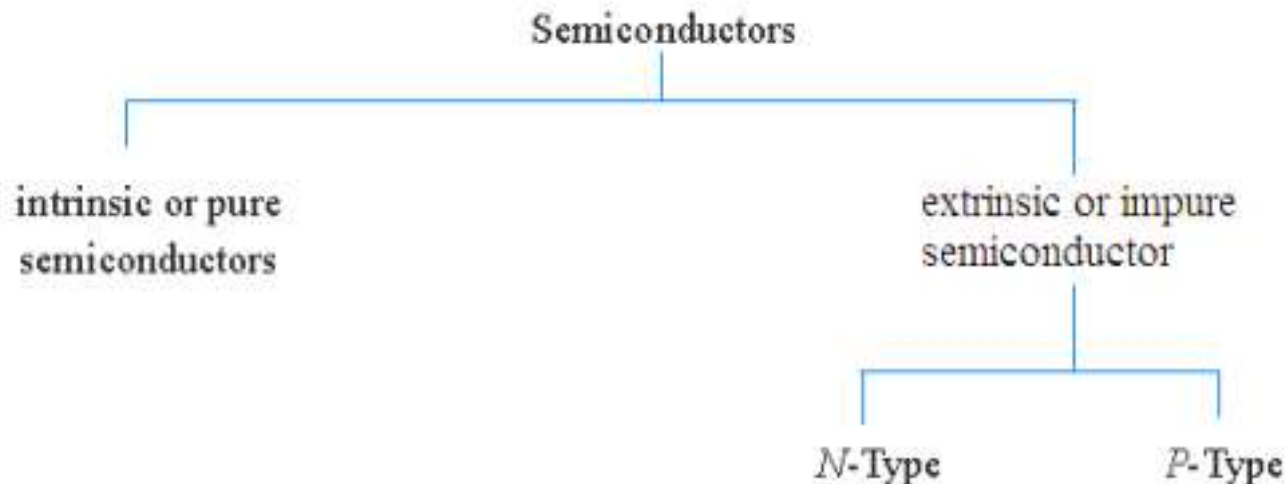
Unit 2: Semiconductors

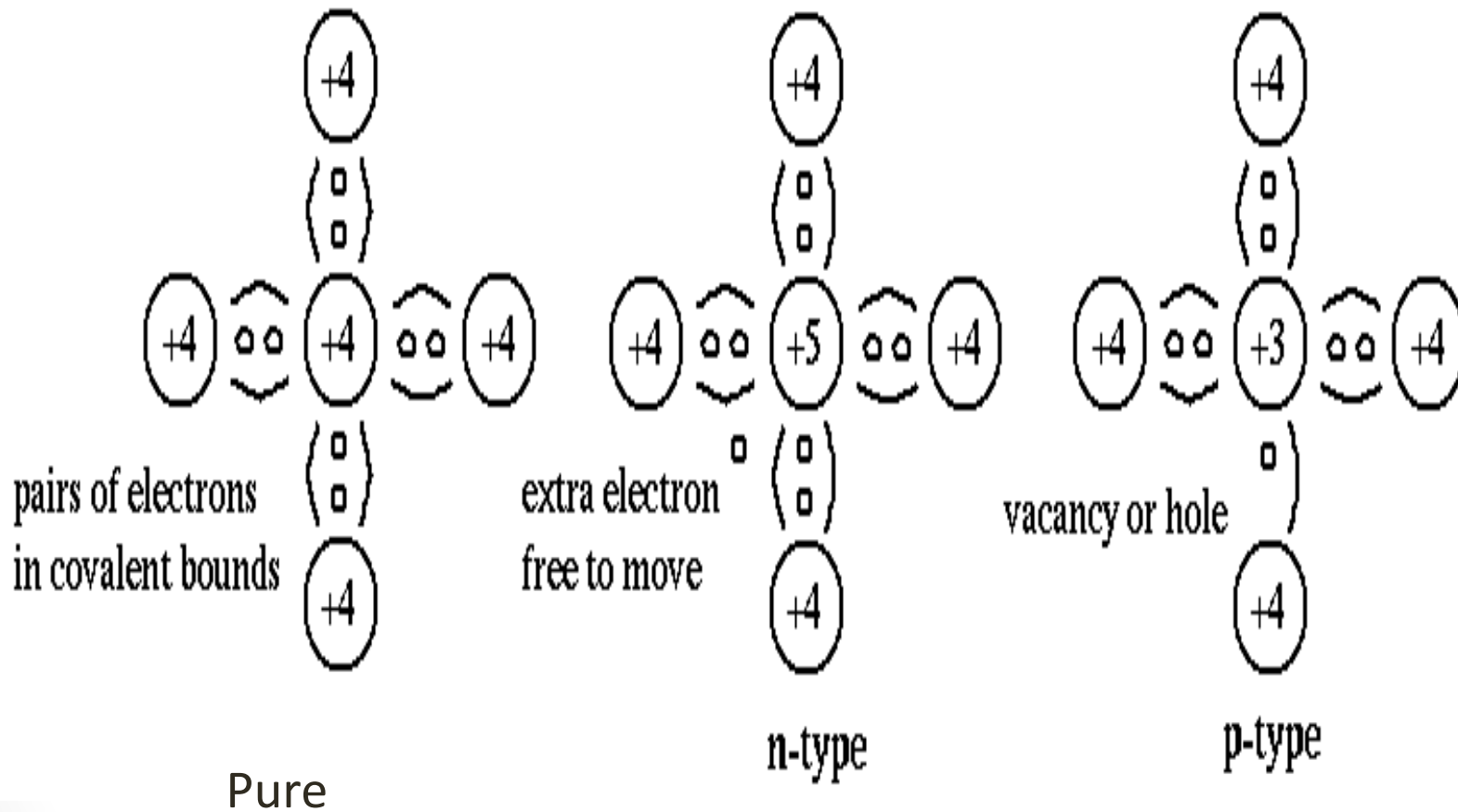
Table of Contents

- Intrinsic and extrinsic semiconductors
- Dependence of Fermi level on carrier-concentration and temperature (equilibrium carrier statistics)
- Carrier generation and recombination
- Carrier transport: diffusion and drift
- p-n junction
- Metal-semiconductor junction (Ohmic and Schottky)
- Semiconductor materials of interest for optoelectronic devices.

Semiconductor

A semiconductor material is one whose electrical properties lie in between those of insulators and good conductors. Examples are: *germanium* and *silicon*. In terms of energy bands, semiconductors can be defined as those materials which have almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 1 eV) separating the two.

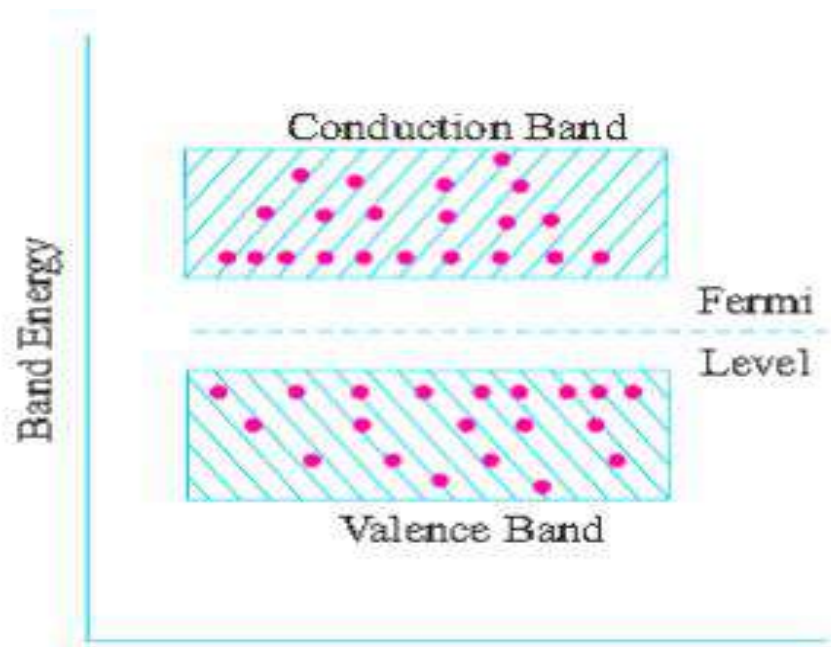




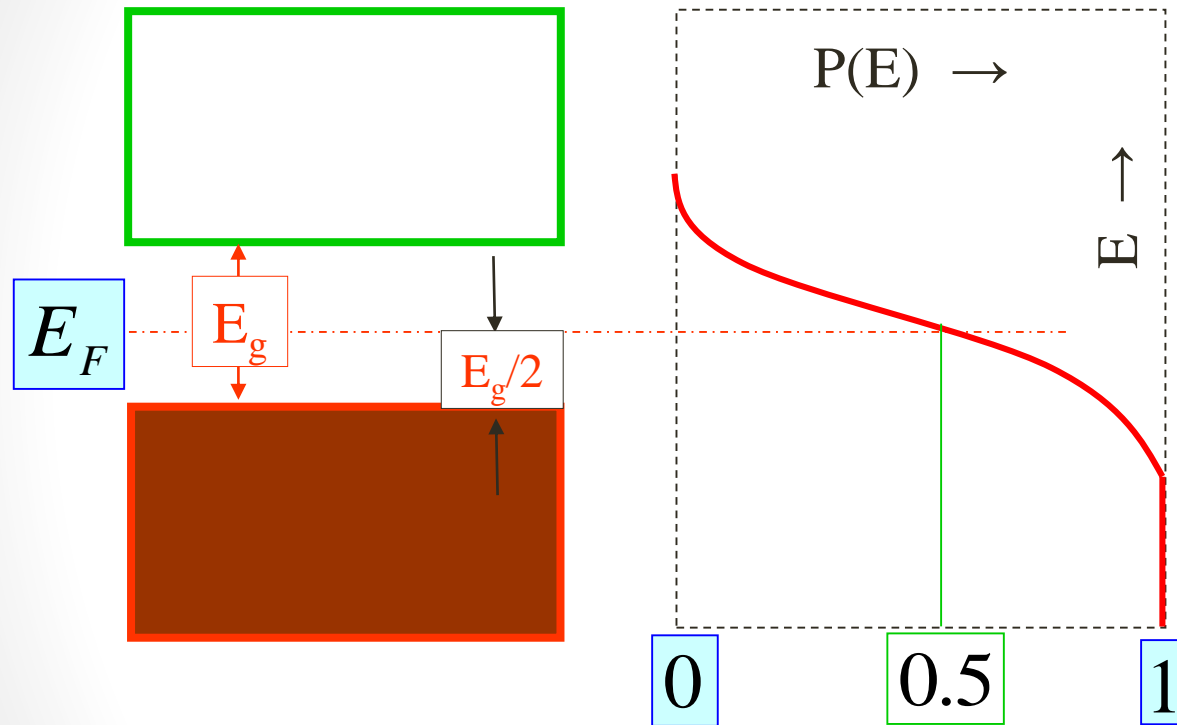
Intrinsic semiconductors

- An intrinsic semiconductor is one which is made of the semiconductor material in its extremely pure form.
- Examples : Si, Ge
- The energy gap is so small that even at ordinary room temperature; there are many electrons which possess sufficient energy to jump across the small energy gap between the valence and the conduction bands.
- Alternatively, an intrinsic semiconductor may be defined as one in which the number of conduction electrons is equal to the number of holes.

- Fermi Level: Fermi level is the term used to describe the top of the collection of electron energy levels at absolute zero temperature. the highest energy **level** which an electron can occupy the valance band at 0k is called **Fermi energy** (E_f).
- Fermi level lies in the mid of forbidden gap in intrinsic semiconductor.



- At zero K very high field strengths ($\sim 10^{10}$ V/m) are required to move an electron from the top of the valence band to the bottom of the conduction band



⇒ Thermal excitation is an easier route.

$$T > 0 K$$

$$P(E) = \frac{1}{1 + \exp\left[\frac{E - E_F}{kT}\right]}$$

$$(E - E_F) = \frac{E_g}{2}$$

$$(E - E_F)_{\text{Silicon}} = \left(\frac{E_g}{2}\right)_{\text{Silicon}} = 0.55 eV$$

$$kT \approx 0.026 eV$$

$$\frac{E - E_F}{kT} \gg 1$$

\Rightarrow Unity in denominator can be ignored

$$P(E) \approx \exp\left[-\frac{E_g}{2kT}\right]$$

$$n_e = N \exp\left[-\frac{E_g}{2kT}\right]$$

- $n_e \rightarrow$ Number of electrons promoted across the gap
(= no. of holes in the valence band)
- $N \rightarrow$ Number of electrons available at the top of the valence band for excitation

Conduction in an intrinsic semiconductor

- Under applied field the electrons (*thermally excited into the conduction band*) can move using the vacant sites in the conduction band
- Holes move in the opposite direction in the valence band
- The conductivity of a semiconductor depends on the concentration of these charge carriers (n_e & n_h)
- Similar to drift velocity of electrons under an applied field in metals in semiconductors the concept of mobility is used to calculate conductivity

$$\text{Mobility} = \frac{\text{drift velocity}}{\text{field gradient}} \quad \left[\frac{m/s}{V/m} \right] = [m^2 / V / s]$$

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

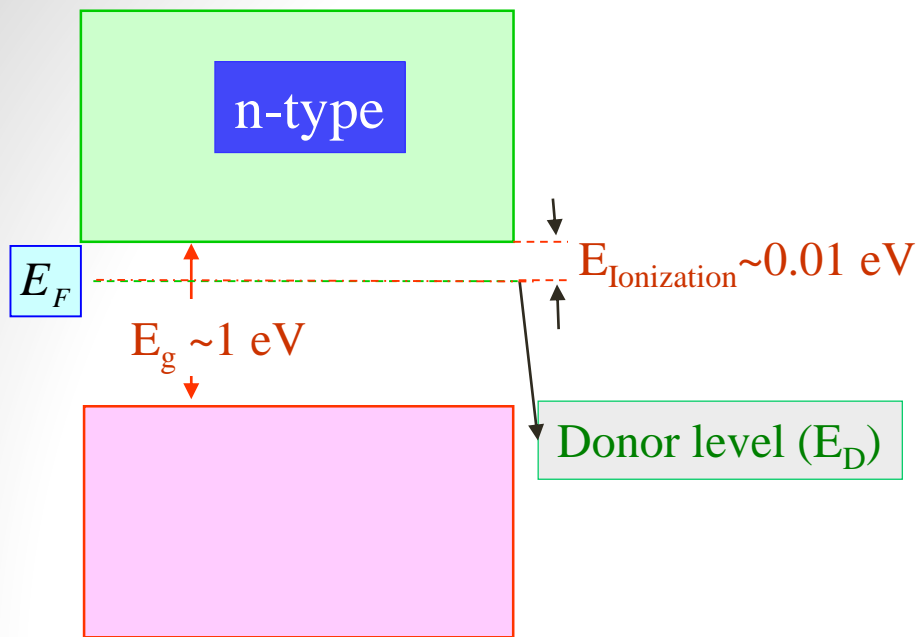
Species	Mobility ($m^2 / V / s$) at RT	
	Si	Ge
Electrons	0.14	0.39
Holes	0.05	0.19

Extrinsic semiconductors

□ The addition of doping elements significantly increases the conductivity of a semiconductor. In fact impurity is accidentally present in semiconductors (even in low concentrations like 1 atom in 10^{12} atoms), which make it extrinsic.

■ Doping of Si

- **V column element** (*P, As, Sb*) → the extra unbonded electron is practically free (*with a radius of motion of $\sim 80 \text{ \AA}$*)
 - ⇒ Energy level near the conduction band
 - ⇒ **n-type** semiconductor
- **III column element** (*Al, Ga, In*) → the extra electron for bonding supplied by a neighbouring Si atom → leaves a hole in Si.
 - ⇒ Energy level near the valence band
 - ⇒ **p-type** semiconductor

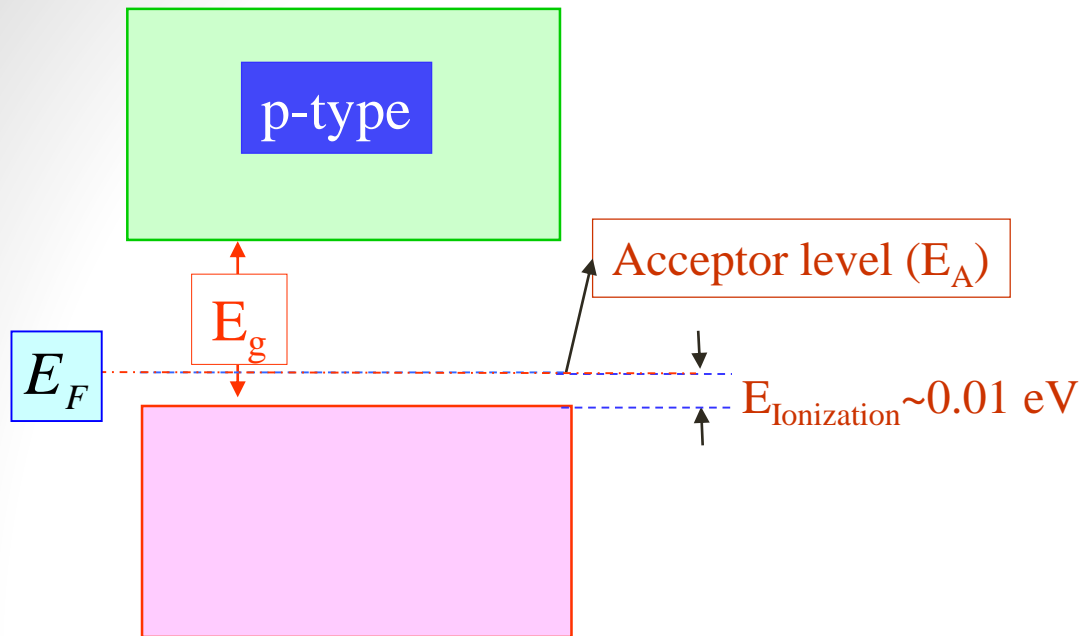


- Ionization Energy \rightarrow Energy required to promote an electron from the Donor level to conduction band.
- $E_{\text{Ionization}} < E_g$
 \Rightarrow even at RT large fraction of the donor electrons are excited into the conduction band.

- ❑ Electrons in the conduction band are the majority charge carriers
- ❑ The fraction of the donor level electrons excited into the conduction band

is much larger than the number of electrons excited from the valence band

- ❑ *Law of mass action:* $(n_e)^{\text{conduction band}} \times (n_h)^{\text{valence band}} = \text{Constant}$
- ❑ The number of holes is very small in an n-type semiconductor
- ❑ \Rightarrow Number of electrons \neq Number of holes



- At zero K the holes are bound to the dopant atom

- ❑ As $T \uparrow$ the holes gain thermal energy and break away from the dopant atom \Rightarrow available for conduction
- ❑ The level of the bound holes are called the acceptor level (which can accept and electron) and acceptor level is close to the valance band
- ❑ Holes are the majority charge carriers
- ❑ Intrinsically excited electrons are small in number
- ❑ Number of electrons \neq Number of holes

N-type Semiconductor

1) Doped with pentavalent atoms. Electrons are majority charge carriers. Holes are minority charge carriers.

2) It gives out electrons, hence, known as donor atoms.

3) Bismuth, Antimony, Arsenic and Phosphorus are the elements used for doping.

P-type Semiconductor

1) Doped with trivalent atoms. Holes are majority charge carriers. Electrons are minority charge carriers.

2) It accepts electrons, hence, known as acceptor atoms.

3) Aluminum, Gallium, Indium and Boron are the elements used for doping.

Conduction Mechanisms in Semiconductors

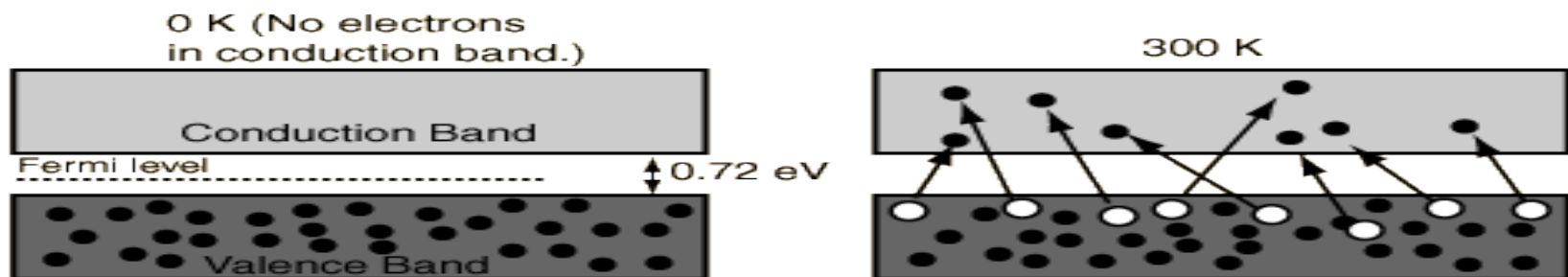
Concentration Laws in Intrinsic and Extrinsic Semiconductors

✓ we study last lecture about intrinsic and extrinsic semiconductors in general.

The general law (charge conservation law) in semiconductors in terms of concentration is

$$np = n_i^2$$

n electron concentration & p hole concentration



charge carrier concentration in intrinsic semiconductors

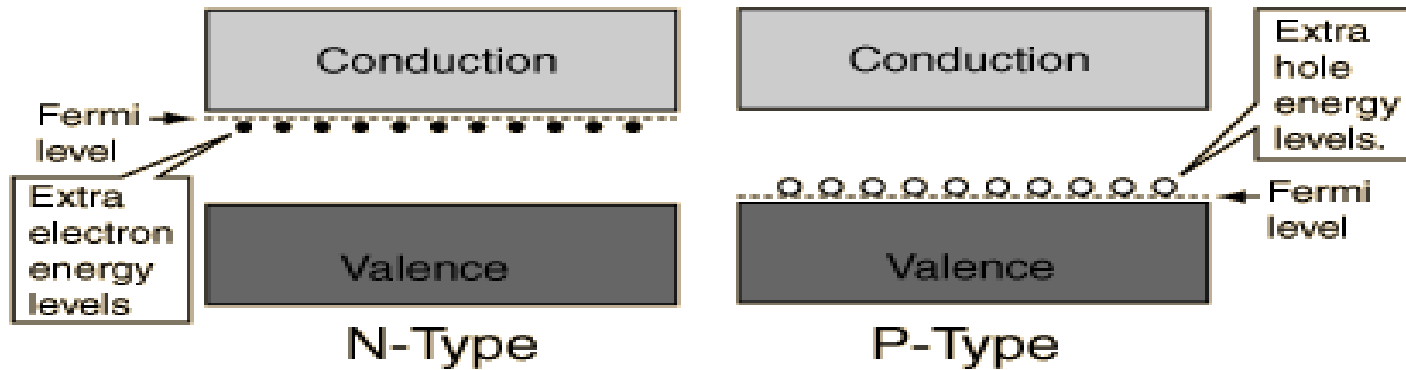
$$n = p = n_i$$

or we can say

$$n_i = p_i$$

n_i is intrinsic charge carrier concentration (whether electrons or holes)

Concentration Laws in Intrinsic and Extrinsic Semiconductors



$$P_{n\text{-type}} < n_{n\text{-type}}$$

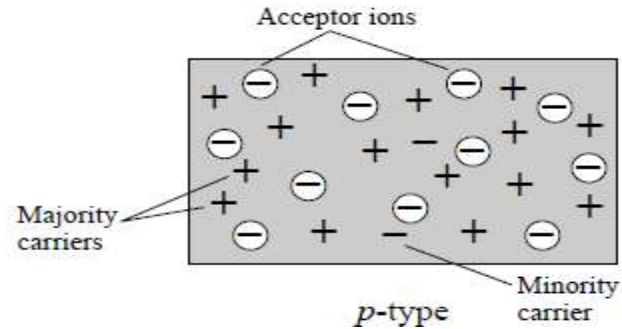
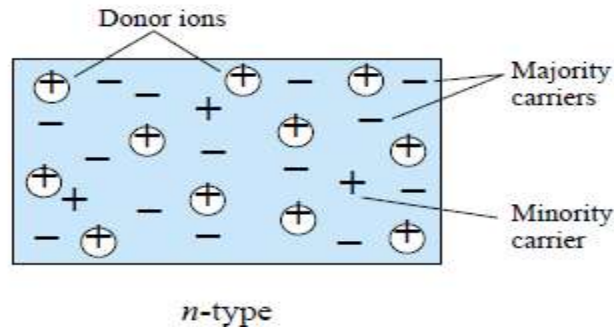
$$P_{p\text{-type}} > n_{p\text{-type}}$$

where n_n is electron concentration in n-type which is the majority carrier { the most number carrier in a semiconductor) & p_n is hole concentration in n-type which is the minority carrier { the fewest number carrier in a semiconductor)

the opposite, n_p is electron concentration in p-type which is the minority carrier p_p is hole concentration in p-type which is the majority carrier .

(concentration is a number per volume unit) (what is unit of concentration)

Concentration Laws in Intrinsic and Extrinsic Semiconductors



N_D^+ is positive ionized donor concentration (such as Sb^+)

N_A^- is negative ionized acceptor concentration (such as B^-)

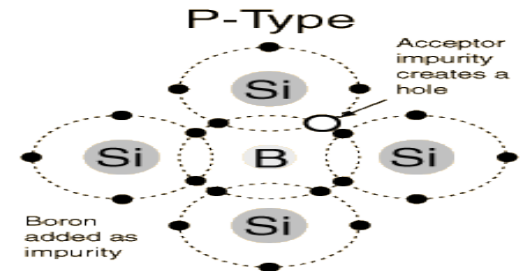
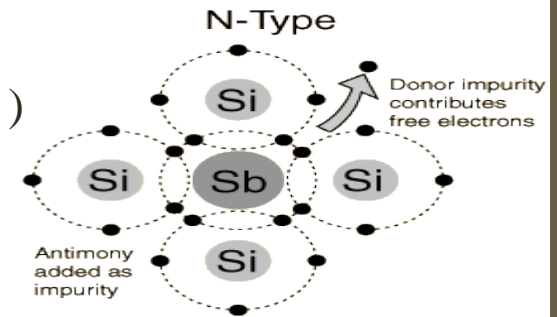
N_D is donor atoms concentration

N_A is acceptor atoms concentration

In room temperature, we obtains complete ionization acceptor and donor atoms.

$$N_D = N_D^+$$

$$N_A = N_A^-$$



(concentration is a number per volume unit) (what is unit of concentration)

Concentration Laws in Intrinsic and Extrinsic Semiconductors

Here also, we apply the general law of concentration in extrinsic semiconductors (charge conservation law)

$$n_n p_n = n_i^2 = n_p p_p$$

❖ charge carrier concentration in n-type

It will be $N_D \gg \gg N_A$ also $N_D \gg \gg n_i$

$$n_n \approx N_D \quad p_n \approx \frac{n_i^2}{N_D}$$

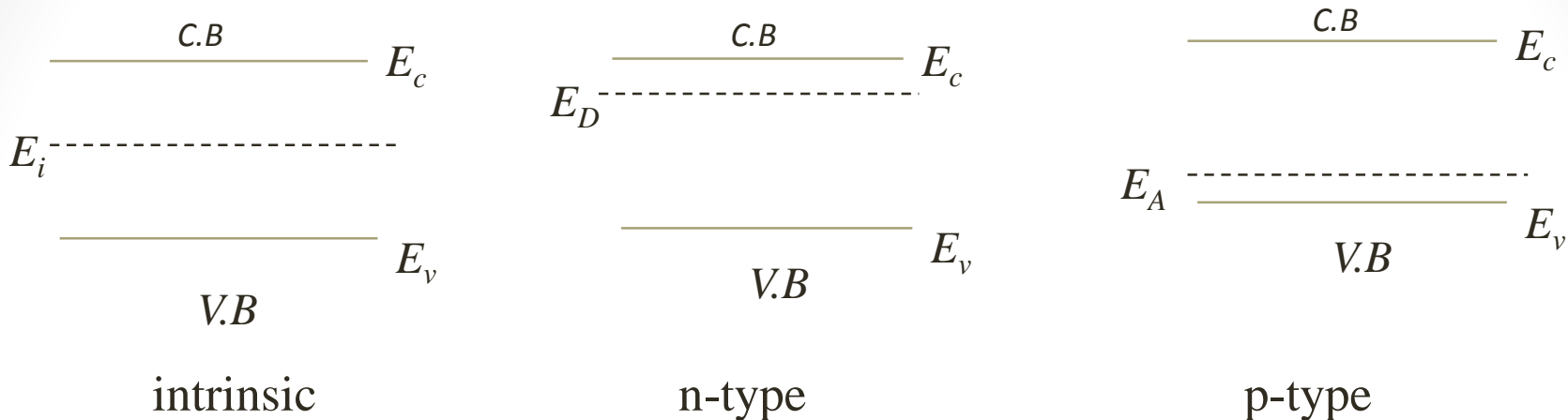
❖ charge carrier concentration in p-type

It will be $N_A \gg \gg N_D$ also $N_A \gg \gg n_i$

$$p_p \approx N_A \quad n_p \approx \frac{n_i^2}{N_A}$$

Energy laws and Fermi level in intrinsic and extrinsic semiconductors

=



acceptor level (E_A) & donor level (E_D) & intrinsic level (E_i)

E_c is for conduction band & E_v is for valence band

Fermi level (E_f) location depend on type of semiconductors is intrinsic or extrinsic (n-type or p-type)

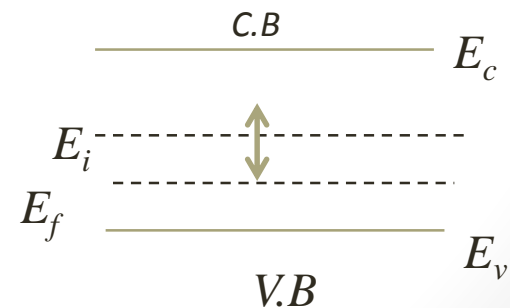
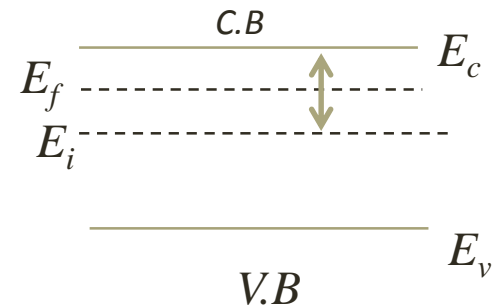
There also equations connect carrier concentration by Fermi level at thermal equilibrium condition

From them, we obtains $p = n_i e^{(E_i - E_f)/kT}$ $n = n_i e^{(E_f - E_i)/kT}$

❖ Therefore in n-type $E_f - E_i = kT \ln\left(\frac{n}{n_i}\right) = -kT \ln\left(\frac{p}{n_i}\right)$

❖ & in p-type $E_f - E_i = kT \ln\left(\frac{n_n}{n_i}\right) = kT \ln\left(\frac{N_D}{n_i}\right)$

$E_f - E_i = -kT \ln\left(\frac{p_p}{n_i}\right) = -kT \ln\left(\frac{N_A}{n_i}\right)$



From last equations, we note how location of Fermi level dependence on carrier concentrations

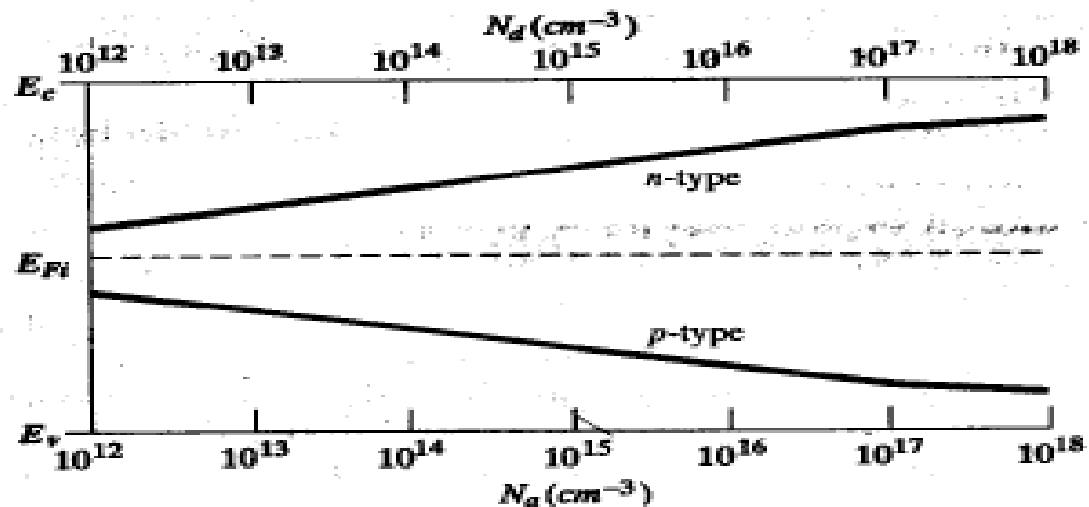
n-type

$$E_f - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$$

p-type

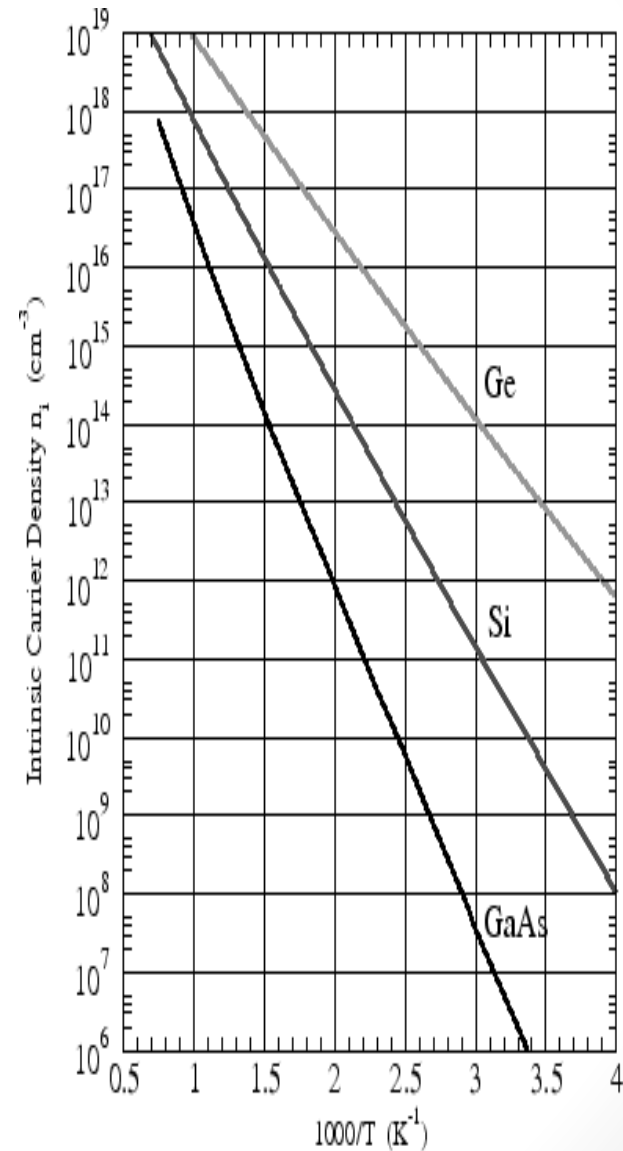
$$E_f - E_i = -kT \ln\left(\frac{N_A}{n_i}\right)$$

This equations applied only for condition of Fermi level is away from conduction and valence band with $3KT$

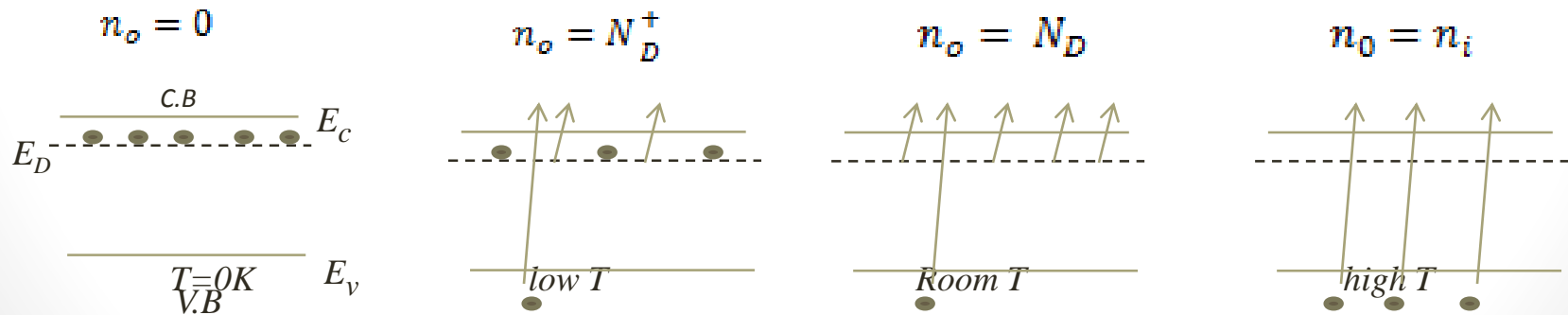
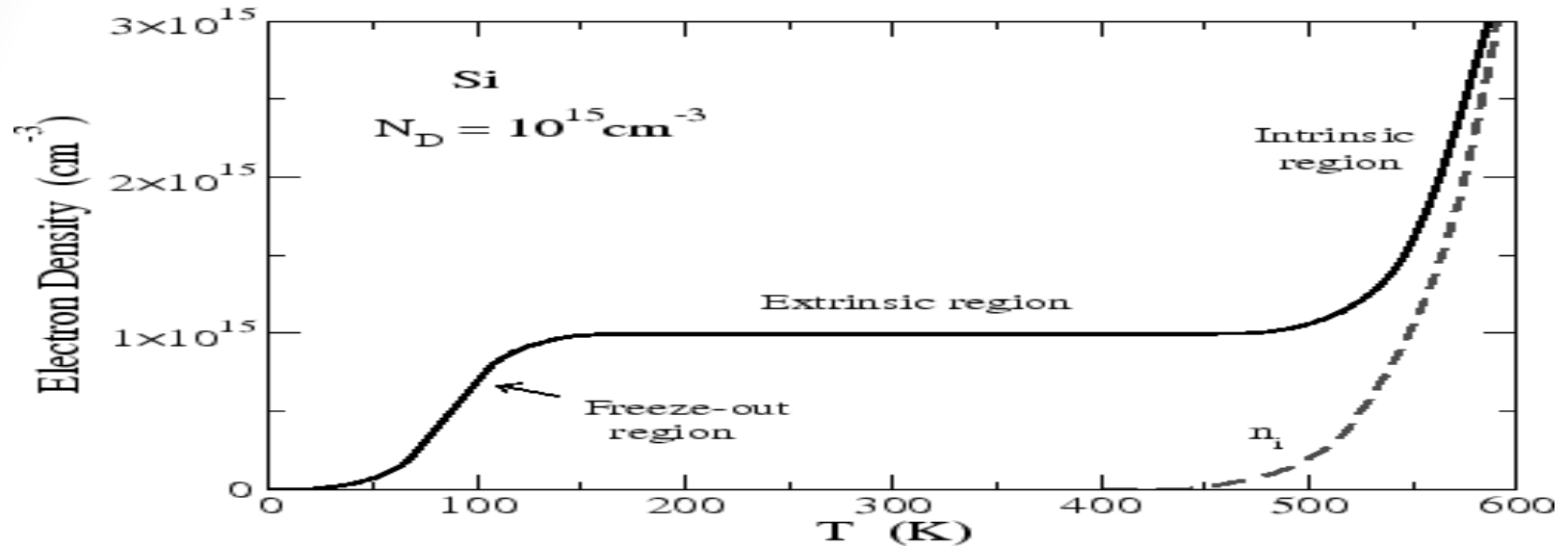


□ Temperature dependence of carrier concentrations :

Also from last equations about concentration, we note not only Fermi level location changes when carrier concentration changes, but also temperature changes when carrier concentration changes whether intrinsic or extrinsic semiconductors. The figure is example of dependence carrier concentration on temperature in intrinsic semiconductors. The relation here is proportional exponential



Example in n-type (semiconductors Si doping with P)



□ Non – equilibrium and Excess Carriers

Our discussion until now is about equilibrium condition, we supposed to symbolize the electron and hole concentration respectively n_o & P_o .

What about non-equilibrium condition?? The simple type is low- level injection which happened when the disorder occurs in electrons concentrations at conduction band or disorder occurs in holes concentrations at valence band less than the concentrations of majority carriers.

N-type $\Delta p \ll n_o$ $n \approx n_o$

P-type $\Delta n \ll p_o$ $p \approx p_o$

n, p are carrier concentrations in any circumstance

$$\Delta n = n - n_o \quad \Delta p = p - p_o$$

Express of disorder of concentrations from equilibrium values. If there was a negative decrease and if there is a positive increase

□ Non – equilibrium and Quasi – Fermi levels

At equilibrium condition, we can locate Fermi level from equilibrium carrier concentration.

$$n_0 = n_i e^{(E_f - E_i)/kT} \quad p_0 = n_i e^{(E_i - E_f)/kT}$$

$$E_f - E_i = kT \ln\left(\frac{n_0}{n_i}\right) = -kT \ln\left(\frac{p_0}{n_i}\right)$$

At non equilibrium condition, we can locate Fermi level from non equilibrium carrier concentration

$$n = n_0 + \Delta n \quad p = p_0 + \Delta p$$

$$n = n_i e^{(F_n - E_i)/kT} \quad p = n_i e^{(E_i - F_p)/kT}$$

$$F_n = E_i + kT \ln\left(\frac{n}{n_i}\right) \quad F_p = E_i - kT \ln\left(\frac{p}{n_i}\right)$$

The transfer of current in semiconductors

The current transfer in the extrinsic semiconductor depends on two important factors, namely drift and diffusion

Drift: movement of charge carriers, which is due to the applied electric field,

✓ This movement is regular which opposite of random Brownian motion. As we know, the holes always move in the direction of the electric field opposite of electrons movement. This movement is called drift.

✓ This movement represent by drift velocity v_d . Drift coefficient is called mobility μ .

✓ Mobility for electron and hole respectively μ_n & μ_p . Also, drift velocity for electron and hole respectively v_n & v_p .

✓ Those coefficients connect by electric field.

$$v_e = -\mu_e \mathcal{E}$$

$$v_h = \mu_h \mathcal{E}$$

✓ What matters to us in this factor is its effect at current transfer and therefore the current density

Current density of drift:

Electrons

$$J_{n/drift} = qn\mu_n \mathbf{E}$$

Holes

$$J_{p/drift} = qp\mu_p \mathbf{E}$$

✓ from the relation between the current density, electric field and resistivity or conductivity $J_{drift} = J_{n/drift} + J_{p/drift} = q (n\mu_e + p\mu_h) \mathbf{E}$

✓ The resistivity and conductivity in the semiconductor express the following equations respectively

$$J = \sigma \mathbf{E} = \frac{1}{\rho} \mathbf{E}$$

$$\sigma = q (n\mu_e + p\mu_h) \quad \rho = \frac{1}{q (n\mu_e + p\mu_h)}$$

□ The migration of carriers (electrons or holes) from the most concentration to
= fewest concentration

✓ Diffusion depends on the concentrations and also on random motion (thermal velocity)

✓ Diffusion coefficient for electron and hole, respectively D_n & D_p

✓ Similar to drift, we need to know current density of diffusion

❖ Current density of diffusion:

Electrons
$$J_n(\text{diffusion}) = qD_n \nabla n$$

Holes
$$J_p(\text{diffusion}) = -qD_p \nabla p$$

= **Total current density in the semiconductor:** is the sum of drift and diffusion current density

Electrons $J_n = J_{n/drift} + J_{n/diff} = nq\mu_n\varepsilon + qD_n\nabla n$

Holes $J_p = J_{p/drift} + J_{p/diff} = pq\mu_p\varepsilon - qD_p\nabla p$

Einstein Relation

We can connect drift and diffusion with another relation. Which is a Einstein Relation

Electrons

$$D_n = \mu_n \frac{kT}{q}$$

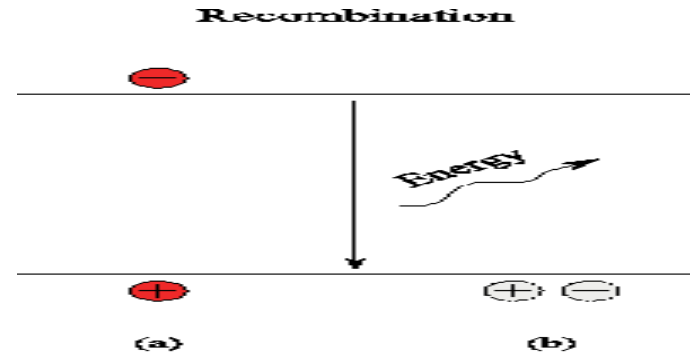
Holes

$$D_p = \mu_p \frac{kT}{q}$$

There are two other processes than drift and diffusion occur in the semiconductor. Which is recombination and generation processes:

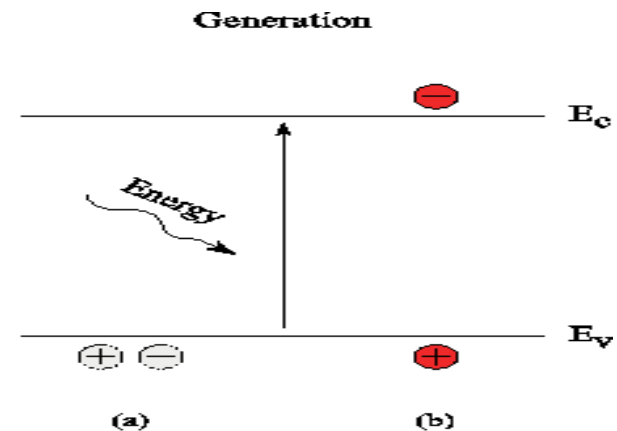
❖ **Recombination:**

The process that electron recombining with hole loses energy and moves from conduction band to valence band



❖ **Generation:**

The process that acquiring energy to generation electron which moves from valence band to conduction band leaving free hole in the valence band; formation of electron-hole pair

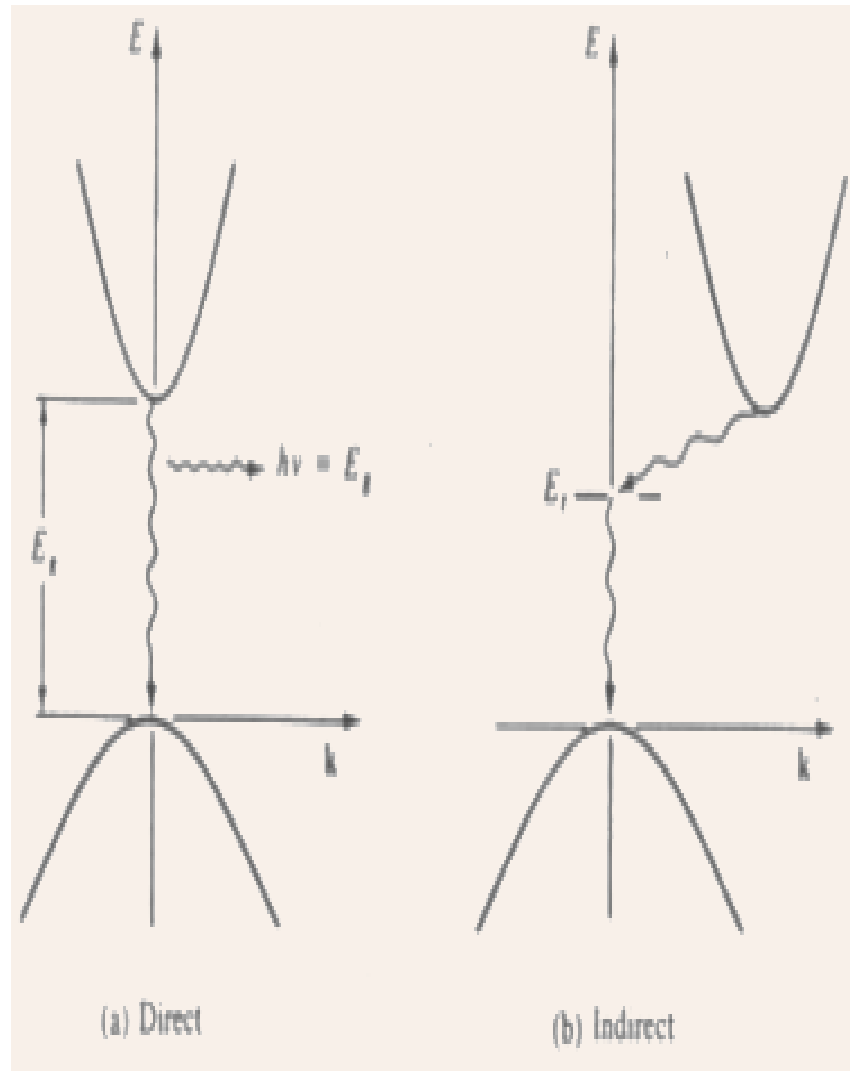


✓ there are number types of this two processes according to given energy (optically, thermally, and kinetically) depending on the circumstances (such as the presence of an electric field) and so on

□ Direct and indirect semiconductors of conduction and valence bands in energy-momentum space

Because we dealt with the subject of recombination and generation, we should clarify that there are two types of semiconductor directly and indirectly.

If the top of valence band and the bottom of conduction band have the same wave number, we called direct semiconductor. However, if they have different wave number, we called indirect semiconductor.

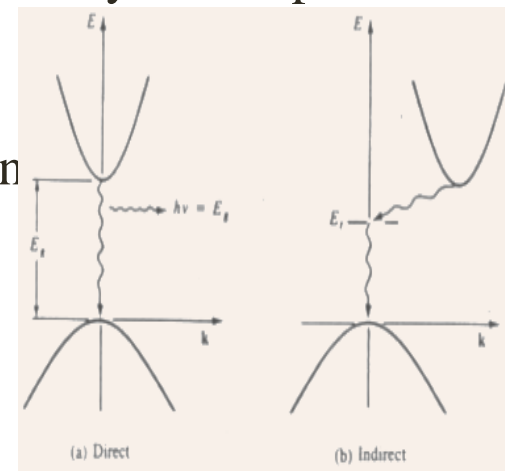


□ Direct and indirect semiconductors of conduction and valence bands in energy-momentum space

recombination or generation direct (band to band) occur in direct semiconductor GaAs, which is famous in the optical diode (LED). The opposite in case of recombination or generation indirect (by recombination and generation centers) occur in indirect semiconductor Si and Ge.

However, the Si more widespread in manufacture diode and transistor than Ge. This is because Ge less stability at temperature, because the valence electrons in it are far from the nucleus and thus their ability to escape easily.

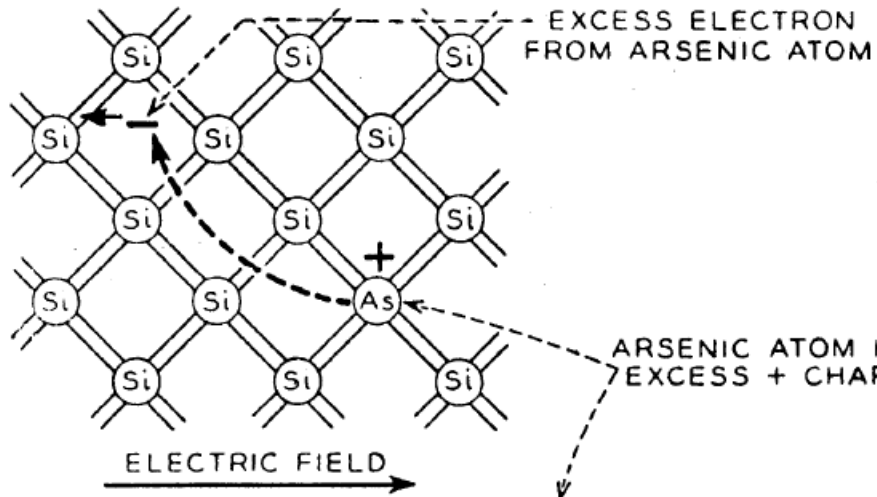
Electrons occupy in most decline valley in conduction band such as ball into the hole. While holes occupy most high peak in valence band such as balloon in a ceiling of a room.



P-N Junction

- P-N junctions are fabricated from a monocrystalline piece of semiconductor with both a P-type and N-type region in proximity at a junction.
- The transfer of electrons from the N side of the junction to holes annihilated on the P side of the junction produces a barrier voltage. This is 0.6 to 0.7 V in silicon, and varies with other semiconductors.
- A forward biased PN junction conducts a current once the barrier voltage is overcome. The external applied potential forces majority carriers toward the junction where recombination takes place, allowing current flow.
- A reverse biased PN junction conducts almost no current. The applied reverse bias attracts majority carriers away from the junction. This increases the thickness of the nonconducting depletion region.
- Reverse biased PN junctions show a temperature dependent reverse leakage current. This is less than a μA in small silicon diodes.

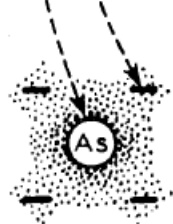
N-type



ARSENIC ATOM HAS EXCESS + CHARGE

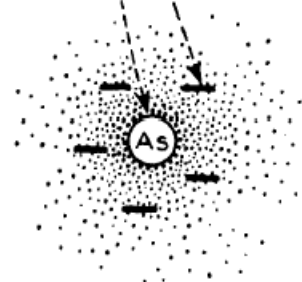
N-TYPE SILICON
(ARSENIC DONORS)

$$+5 - 4 = +1$$



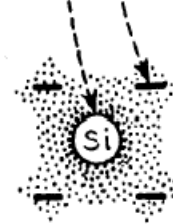
CHARGED
ARSENIC ATOM
IN SILICON CRYSTAL

$$+5 - 5 = 0$$



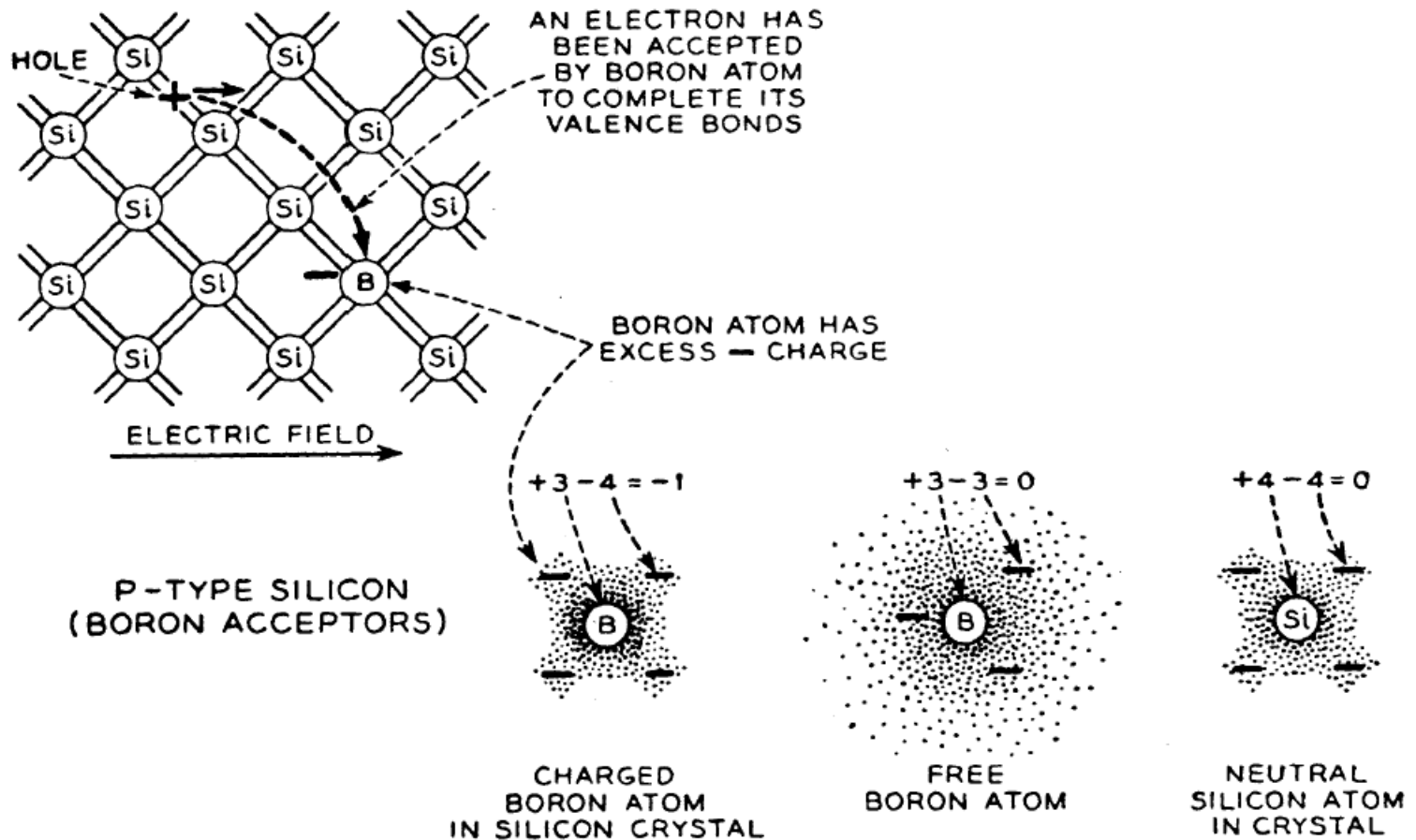
FREE
ARSENIC ATOM

$$+4 - 4 = 0$$

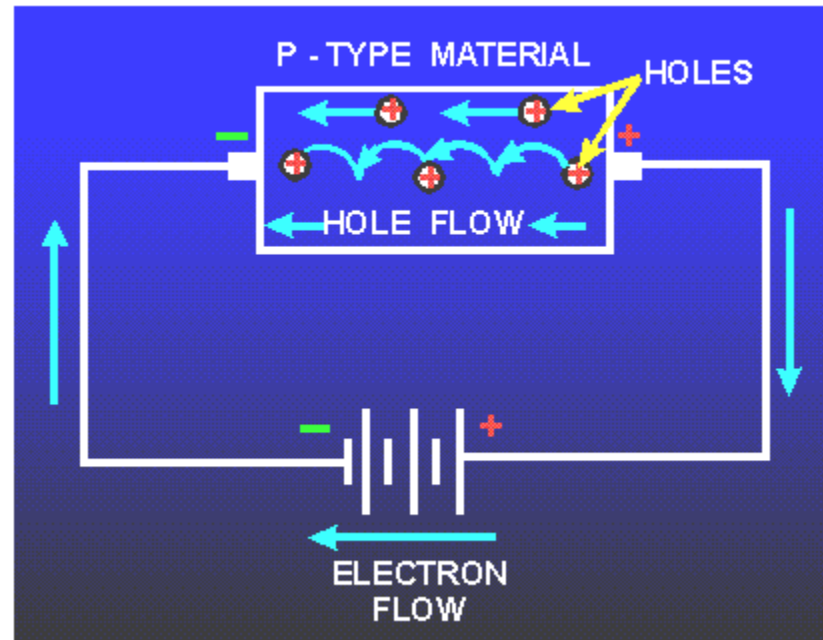
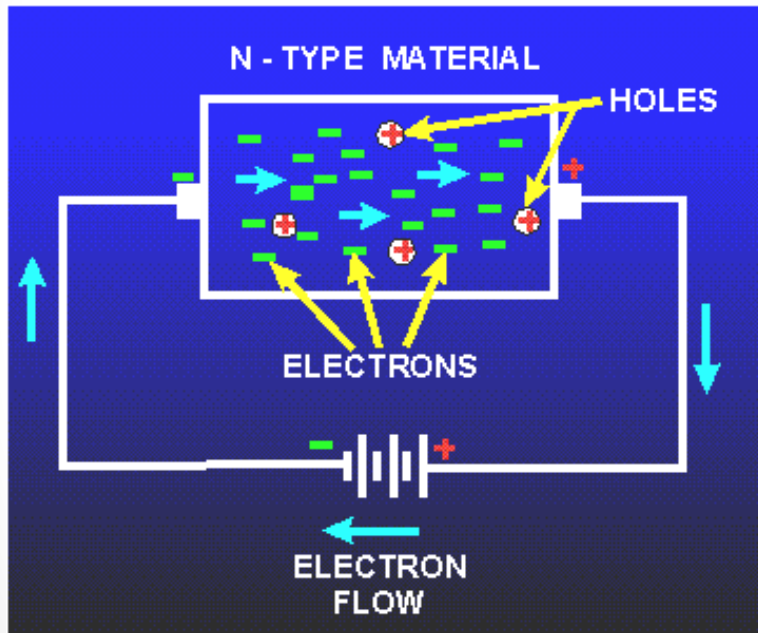


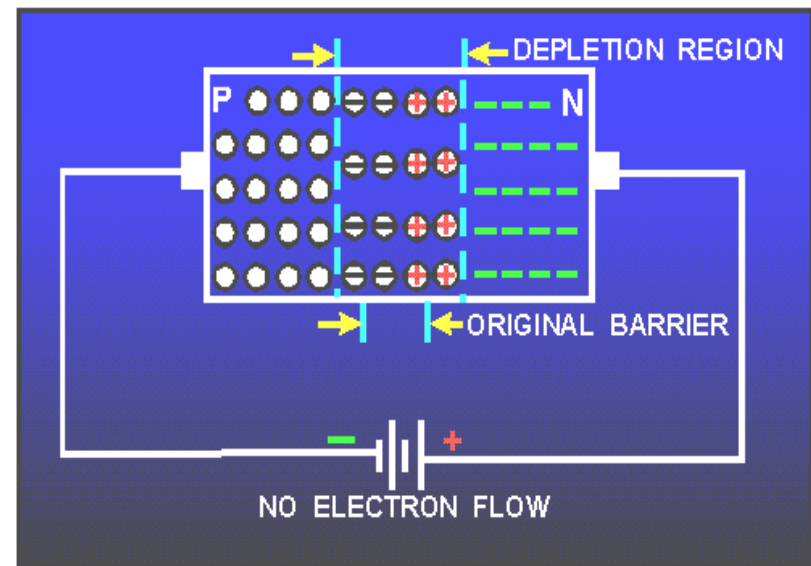
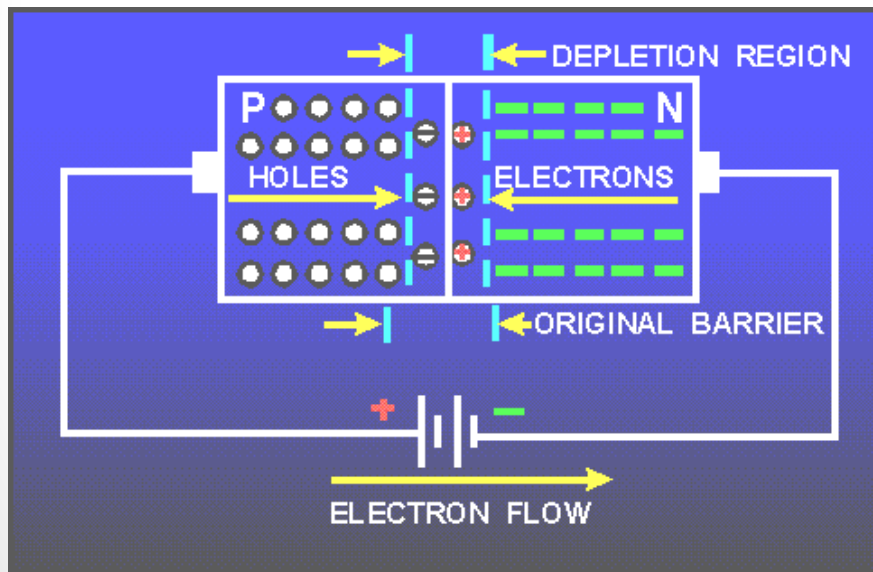
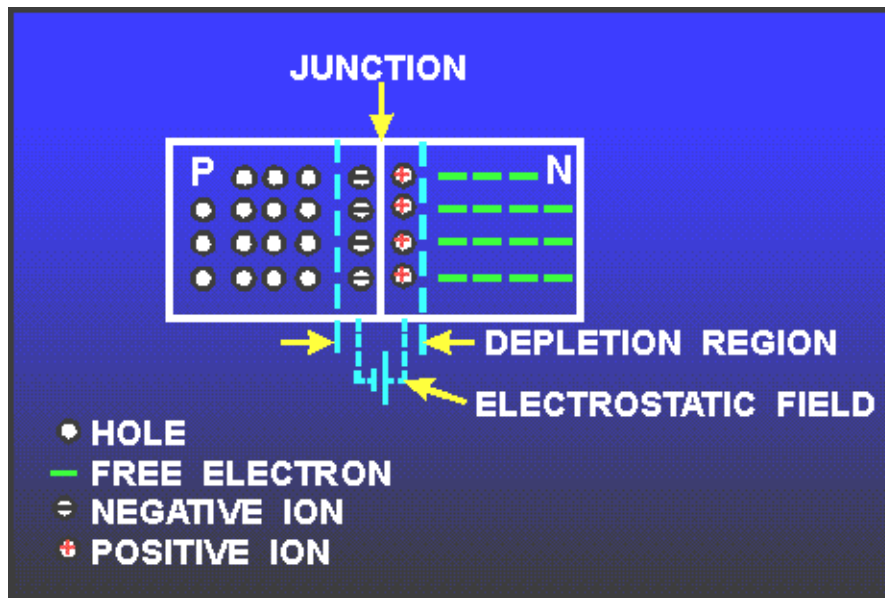
NEUTRAL
SILICON ATOM
IN CRYSTAL

P-type



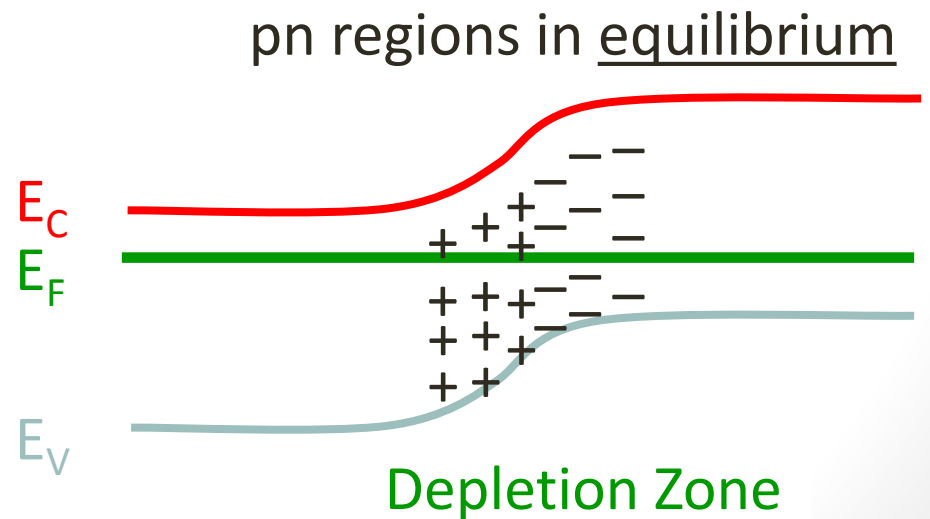
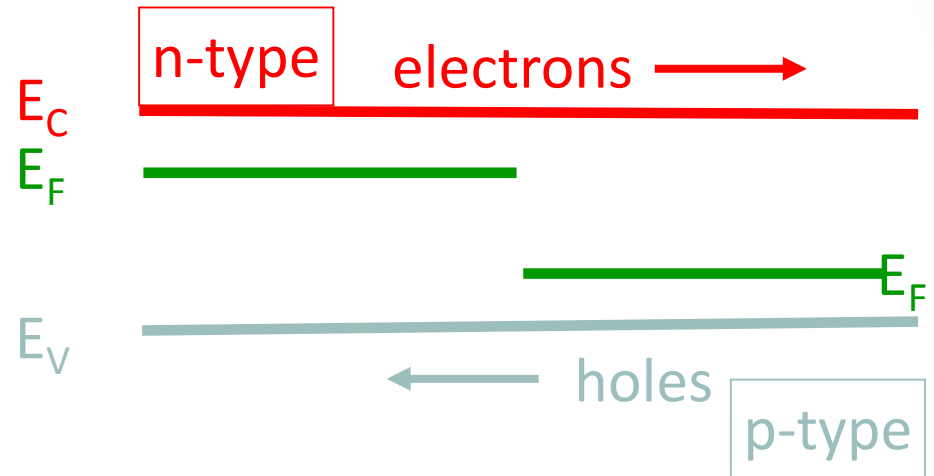
Conduction in p/n-type Semiconductors





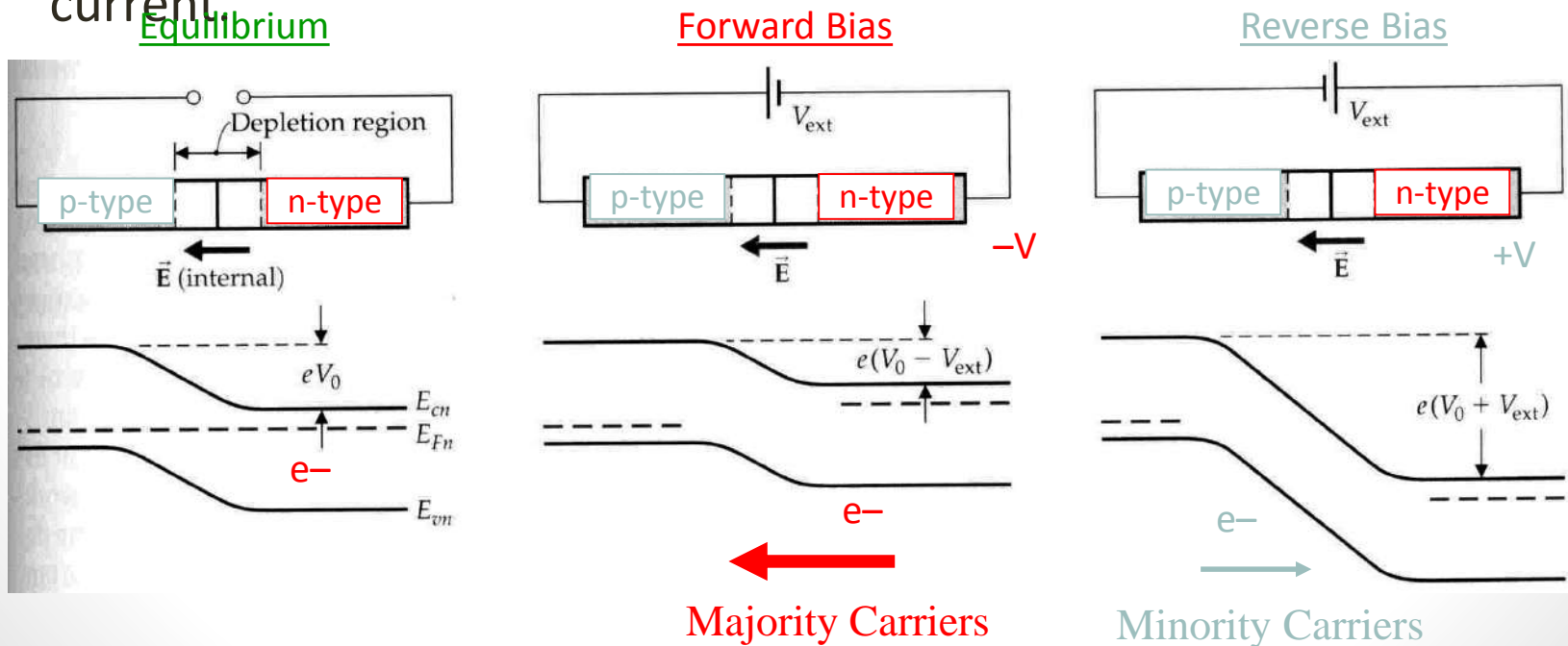
PN Junction: Band Diagram

- Due to diffusion, electrons move from n to p-side and holes from p to n-side.
- Causes depletion zone at junction where immobile charged ion cores remain.
- Results in a built-in electric field (10^3 to 10^5 V/cm), which opposes further diffusion.
- Note: E_F levels are aligned across pn junction under equilibrium.

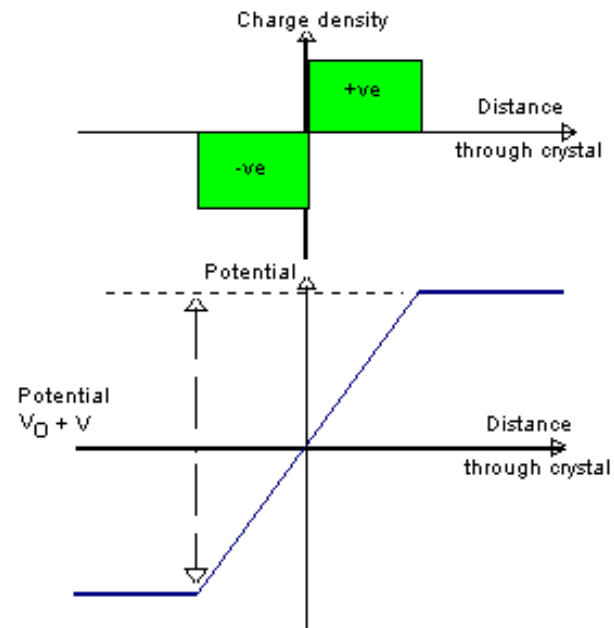
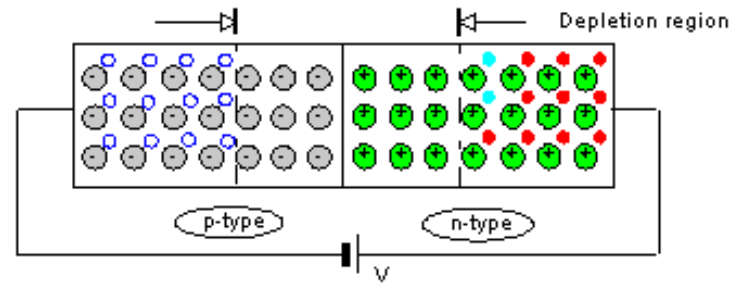
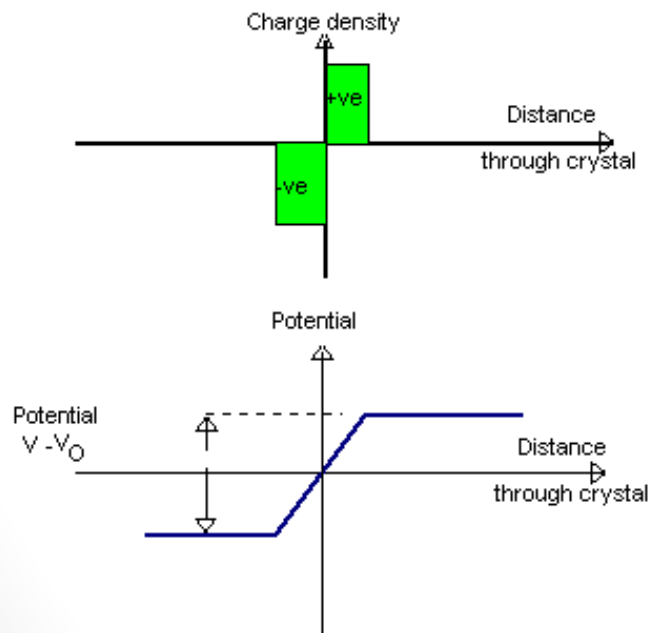
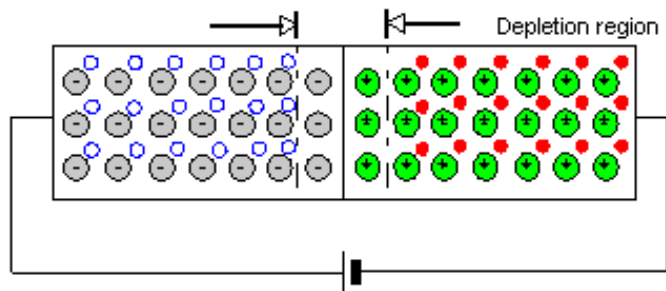


PN Junction: Band Diagram under Bias

- Forward Bias: negative voltage on n-side promotes diffusion of electrons by decreasing built-in junction potential \rightarrow higher current.
- Reverse Bias: positive voltage on n-side inhibits diffusion of electrons by increasing built-in junction potential \rightarrow lower current.



Forward & Reverse Biased

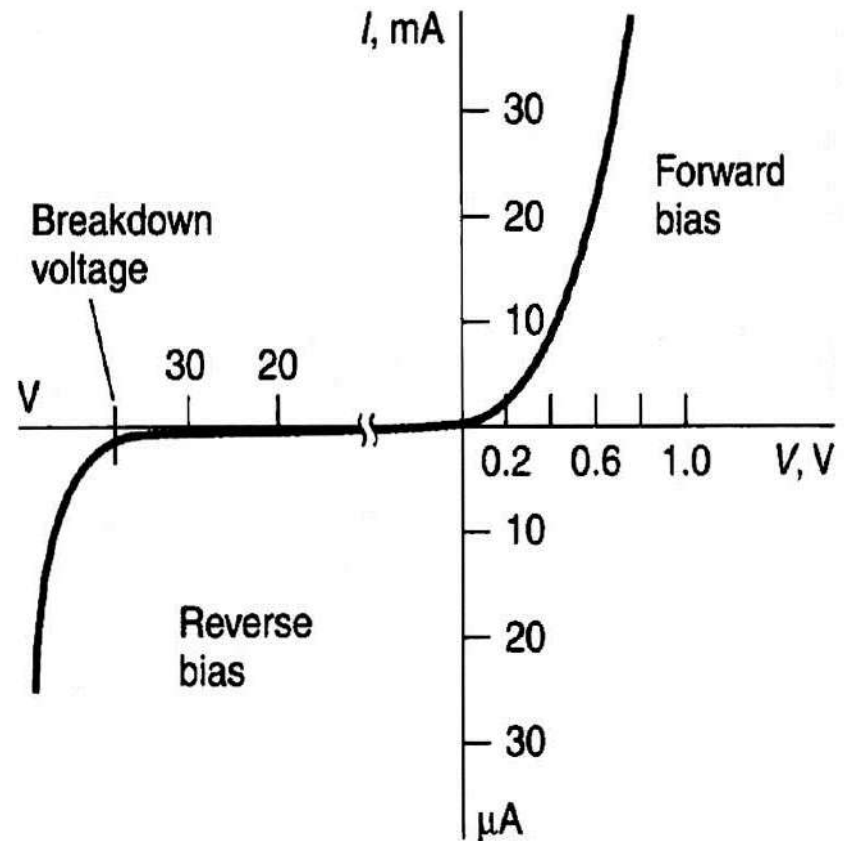


PN Junction: IV Characteristics

- Current-Voltage Relationship

$$I = I_o [e^{eV/kT} - 1]$$

- Forward Bias: current exponentially increases.
- Reverse Bias: low leakage current equal to $\sim I_o$.
- Ability of pn junction to pass current in only one direction is known as “rectifying” behavior.



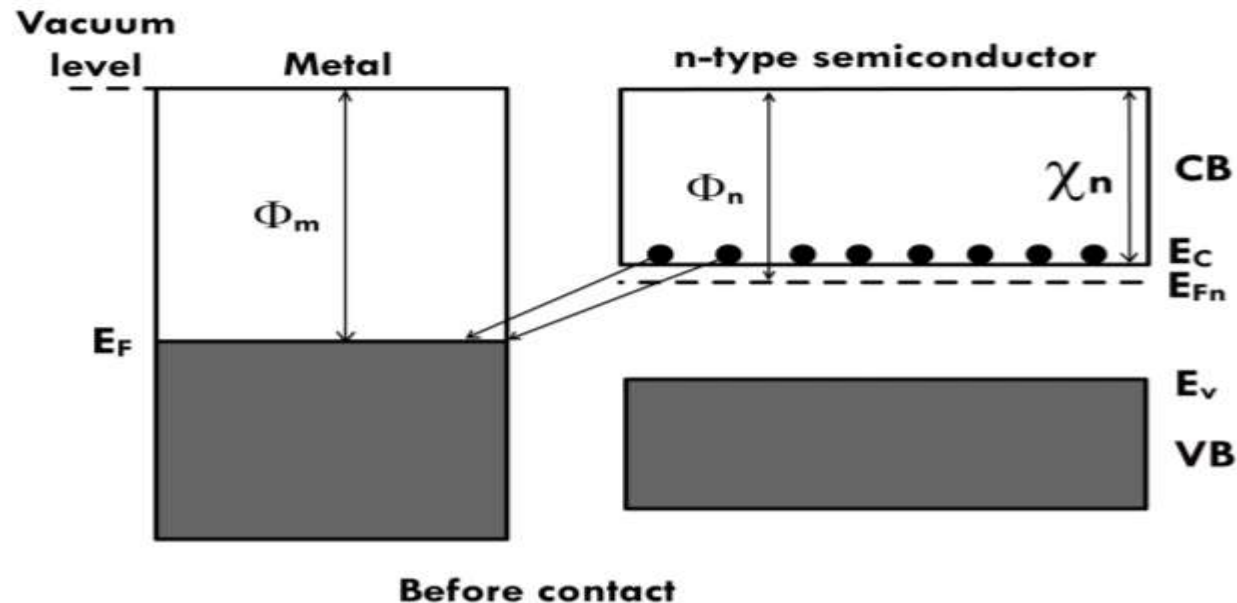
Schottky junctions

When a metal and semiconductor are brought into contact, there are two types of junctions formed depending on the work function of the semiconductor, its relation with the metal

1. Schottky junction : $\phi_m > \phi_{\text{semi}}$

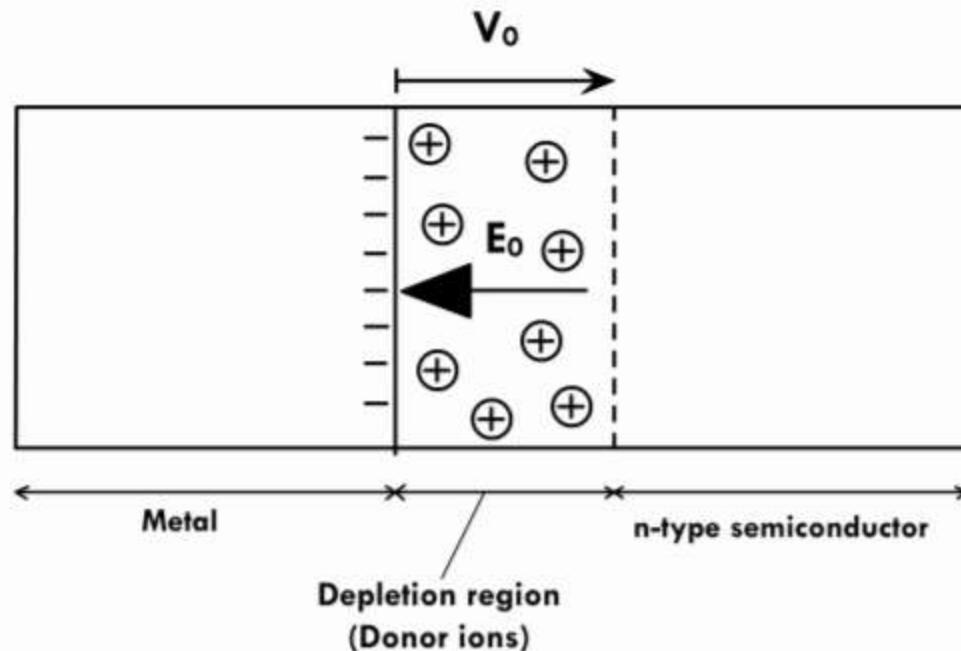
2. Ohmic junction : $\phi_m < \phi_{\text{semi}}$

Consider a junction formed between a metal and n-type semiconductor, as shown in figure below.

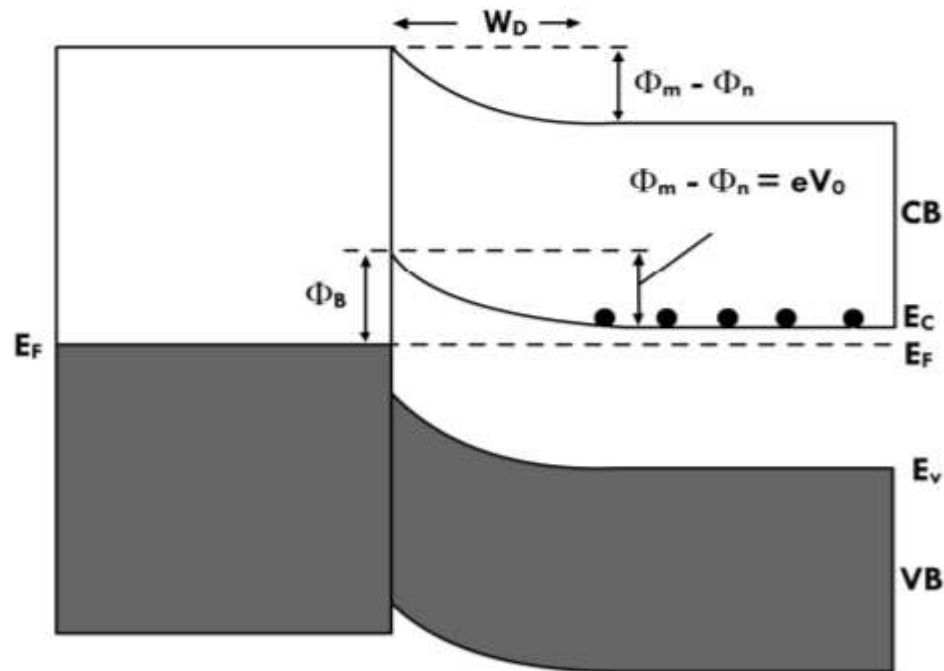


- The Fermi level of the semiconductor is higher (since its work function is lower) than the metal. Similar to a metal-metal junction, when the metal-semiconductor junction is formed the Fermi levels must line up at equilibrium.
- Another way to look at this is that there are electrons in the conduction level of the semiconductor which can move to the empty energy states above the Fermi level of the metal.
- This leaves a positive charge on the semiconductor side and due to the excess electrons, a negative charge on the metal side, shown in figure below, leading to a contact potential.
- When a contact is formed between two metals, the charges reside on the surface. This is due to the high electron density found in metals (typically 10^{22} cm^{-3}).

On the other hand, when a contact is formed between a metal and semiconductor, due to the low charge density on the semiconductor side (typically 10^{17} cm^{-3}) the electrons are removed not only from the surface but also from a certain depth within the semiconductor. This leads to the formation of a depletion region within the semiconductor, shown in figure below.



When a Schottky junction is formed between the metal and semiconductor, the Fermi level lines up and also a positive potential is formed on the semiconductor side. Because the depletion region extends within a certain depth in the semiconductor there is bending of the energy bands on the semiconductor side. Bands bend up in the direction of the electric field (field goes from positive charge to negative charge, opposite of the potential direction). This means the energy bands bend up going from n-type semiconductor to metal, shown in figure below.



There is a built in potential in the Schottky junction, V_0 , and from figure 8 this is given by the difference in work functions.

$$eV_0 = \phi_m - \phi_{\text{semi}}$$

The work function of the metal is a constant while the semiconductor work function depends on the dopant concentration (since this affects the Fermi level position). The contact potential then represents the barrier for the electrons to move from the n-type semiconductor to the metal. Initially, when the junction is formed electrons move to the metal to create the depletion region. The contact potential thus formed prevents further motion of the electrons to the metal. There is also a barrier for electrons to move from metal to semiconductor. This is called the Schottky barrier and denoted by ϕ_B in figure.

$$\phi_B = (\phi_m - \phi_n) + (E_c - E_{Fn}) = \phi_m - \chi_n$$

Biasing

The Schottky junction can be biased by application of an external potential. There are two types of bias:

1. Forward bias - metal is connected to positive terminal and n-type semiconductor connected to negative terminal.
2. Reverse bias - metal is connected to negative terminal and n-type semiconductor connected to positive terminal.

The current flow depends on the type of bias and the amount of applied external potential.

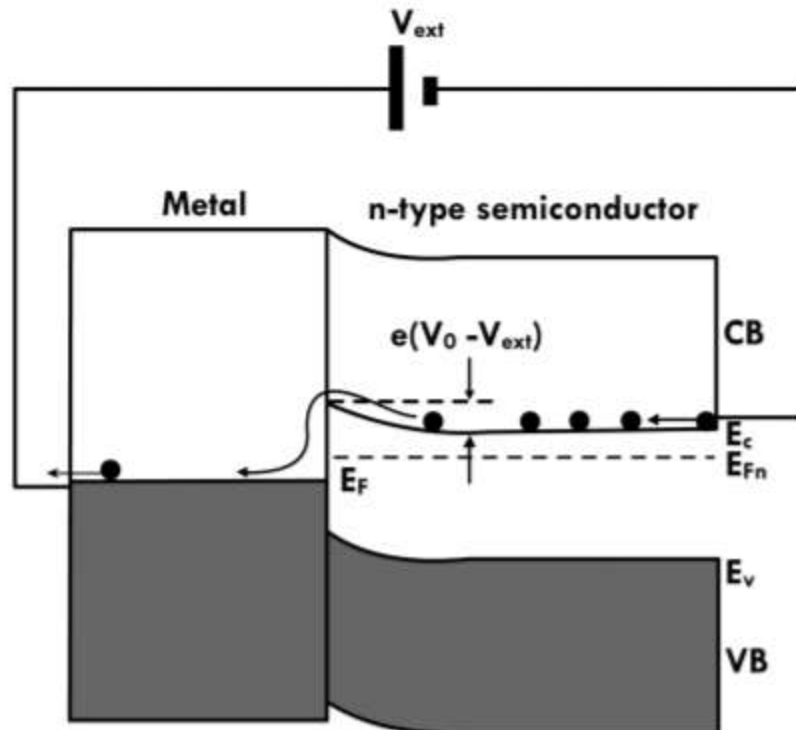
Forward bias

The current in a Schottky diode under forward bias is given by

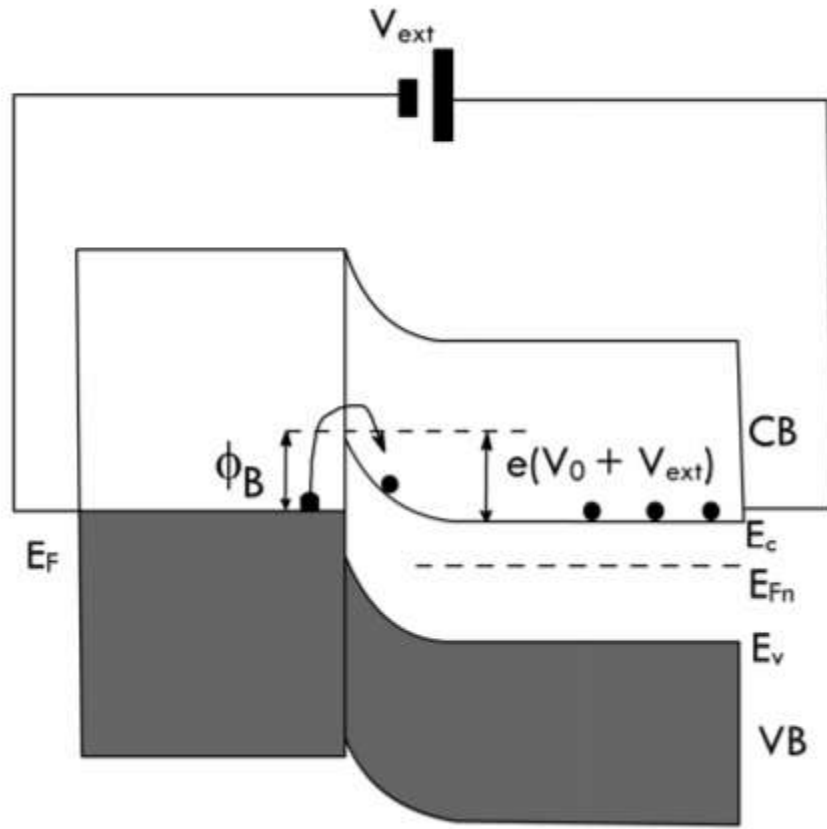
$$J = J_0 \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

where J is the current density for an applied potential of V . J_0 is a constant and depends on the Schottky barrier (ϕ_B) for the system and the expression is

$$J_0 = AT^2 \exp\left(-\frac{\phi_B}{k_B T}\right)$$

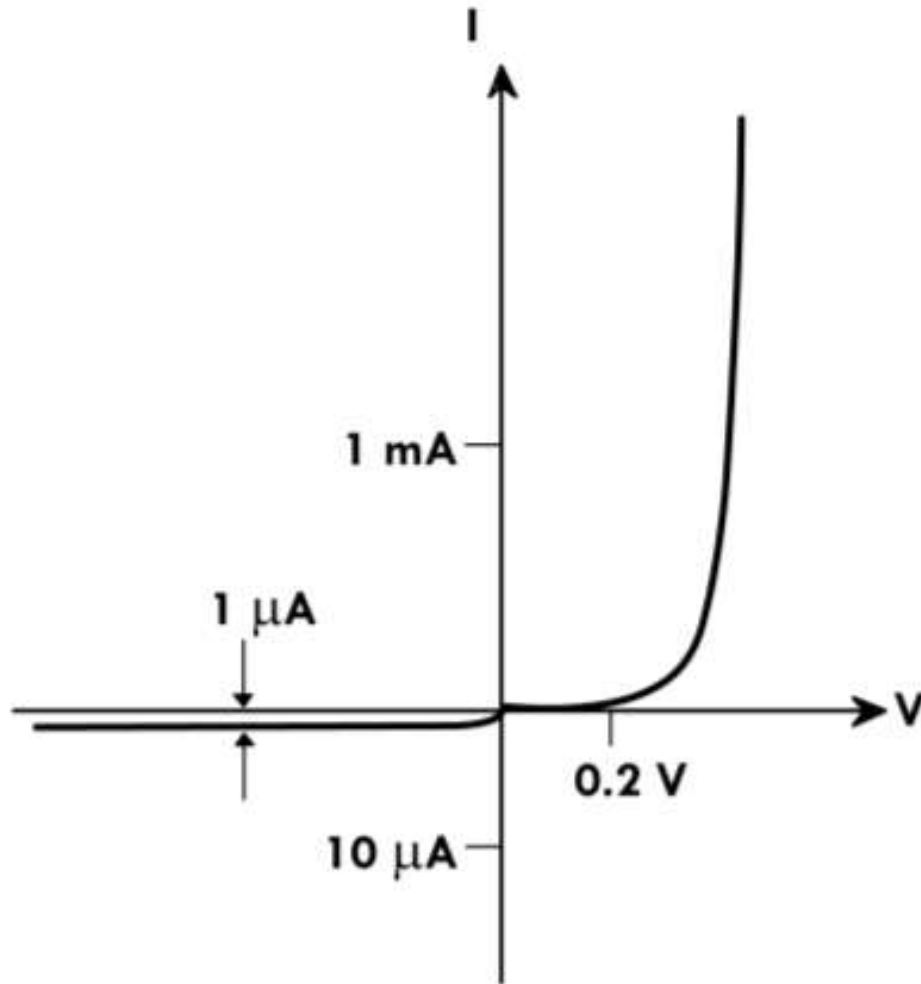


Reverse Bias



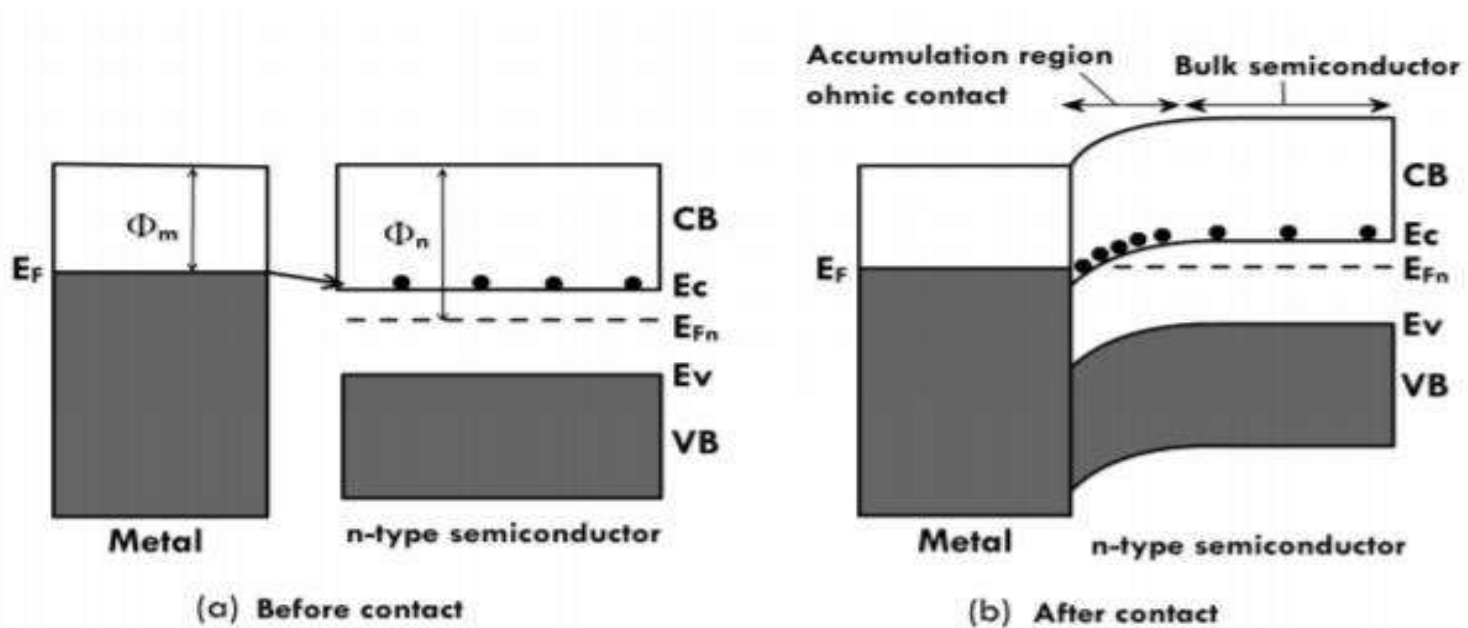
Schottky junction under reverse bias.

$I - V$ characteristics of a Schottky junction



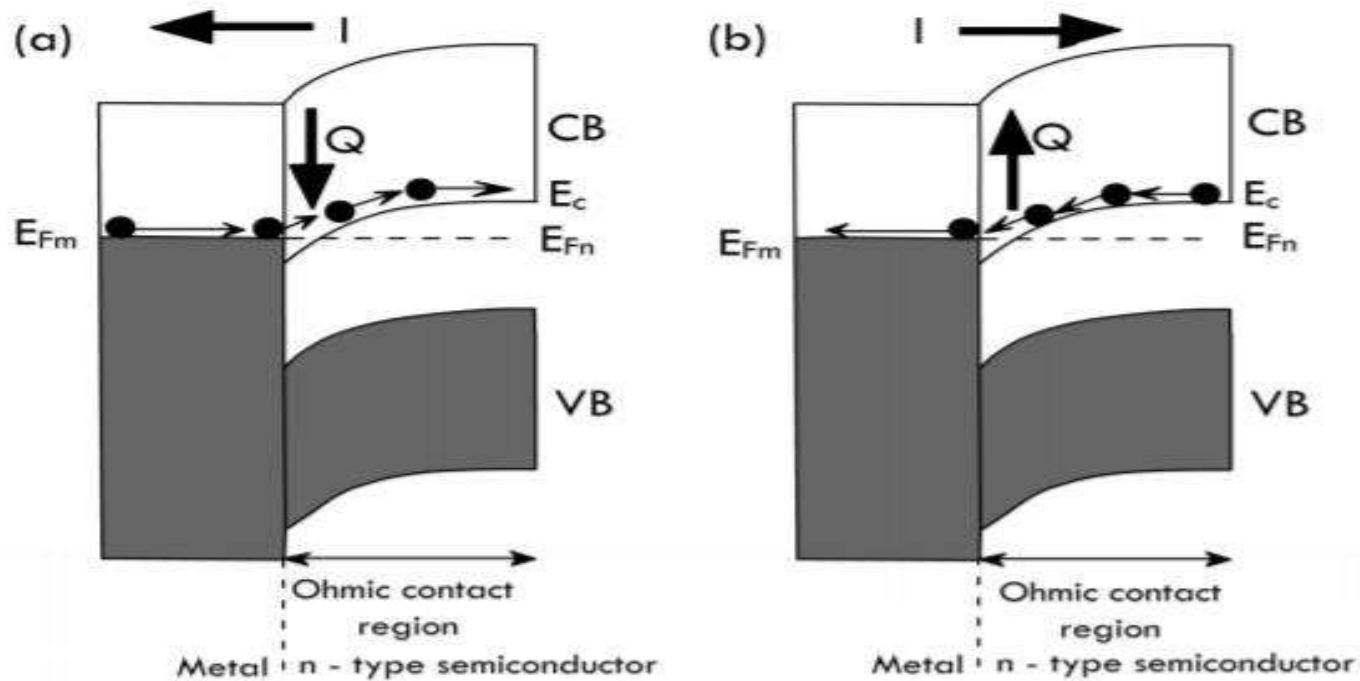
Ohmic Junction

- When the semiconductor has a higher work function the junction formed is called the Ohmic junction. The junction in equilibrium (Fermi levels line up). This is shown in figure below.



- At equilibrium, electrons move from the metal to the empty states in the conduction band so that there is an accumulation region near the interface (on the semiconductor side).

The accumulation region has a higher conductivity than the bulk of the semiconductor due to this higher concentration of electrons. Thus, a Ohmic junction behaves as a resistor conducting in both forward and reverse bias. The resistivity is determined by the bulk resistivity of the semiconductor.

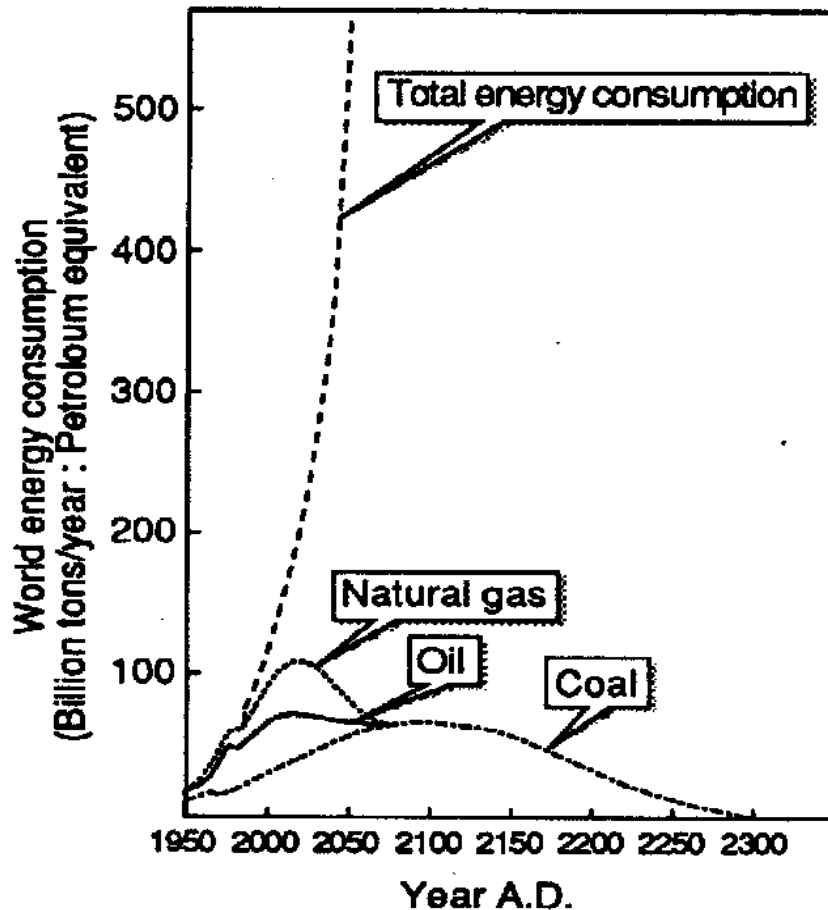


Current flow through an Ohmic junction can lead to heat (a) absorption or (b) release. This depends on the external bias, that determines the direction of heat flow. When electrons move from metal to higher energy levels in the semiconductor heat is absorbed and the reverse happens when electrons flow from semiconductor to metal.

Semiconductor materials of interest for optoelectronic devices

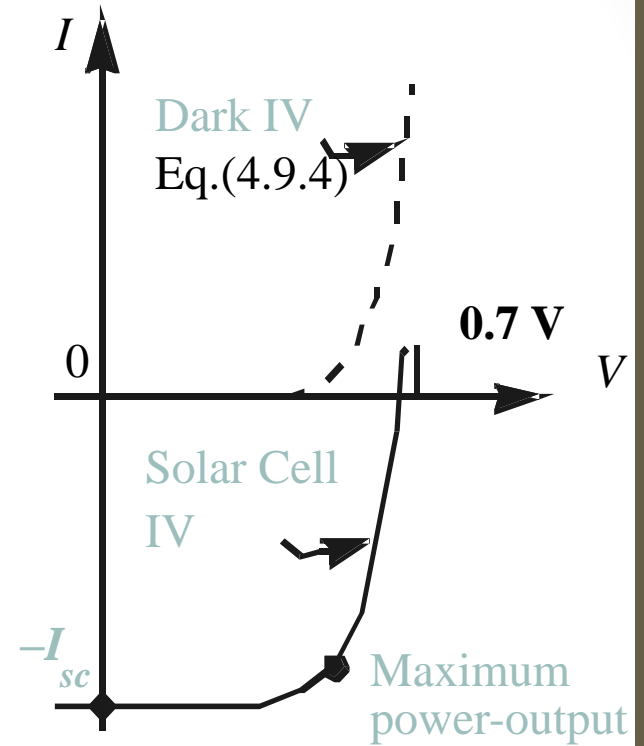
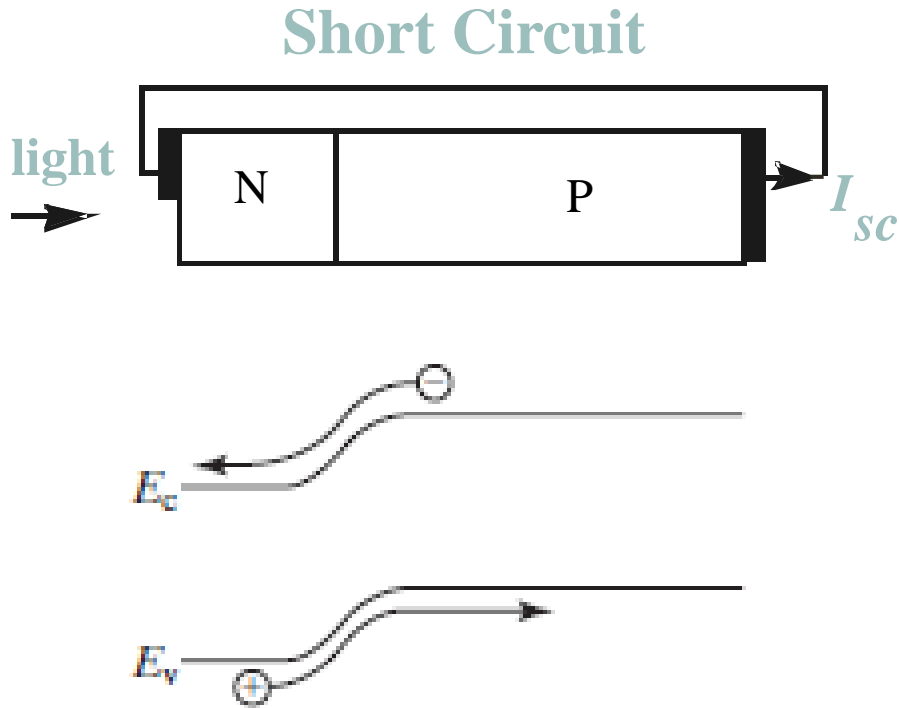
1. Solar Cells
2. LED's
3. Solid state lighting
4. Diode Lasers
5. Photo Diodes

Solar Cells



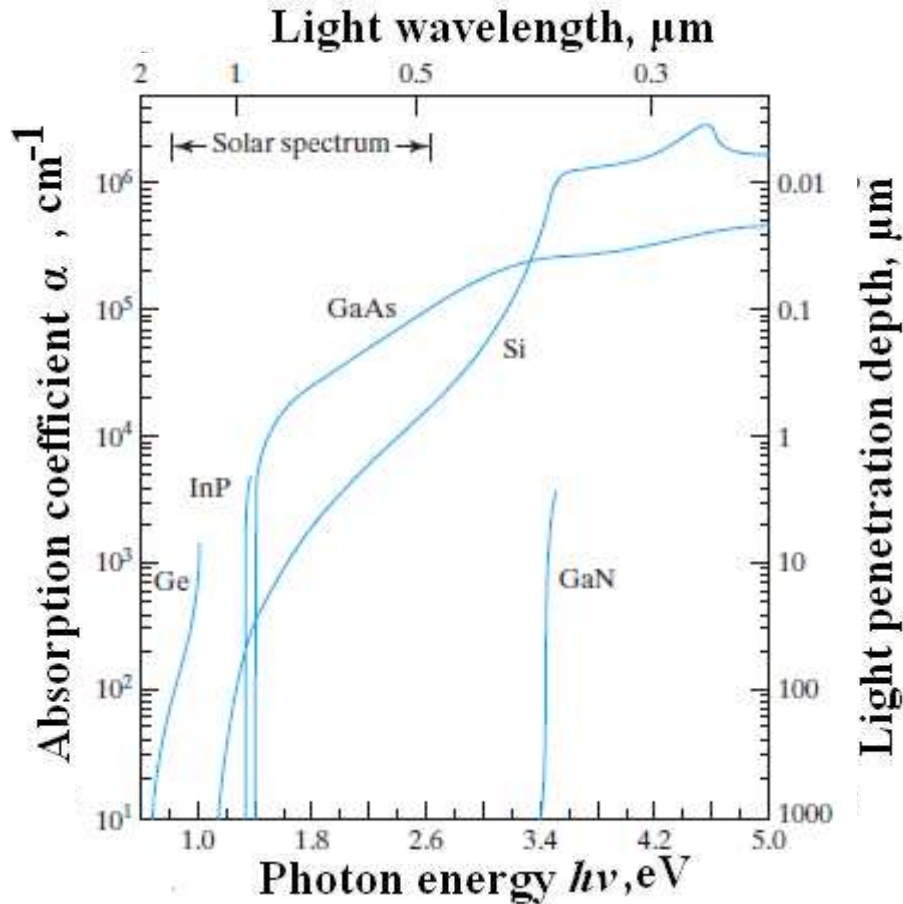
- *Solar Cells* is also known as *photovoltaic cells*.
- Converts sunlight to electricity with 10-30% conversion efficiency.
- 1 m² solar cell generate about 150 W peak or 25 W continuous power.
- Low cost and high efficiency are needed for wide deployment.

Solar Cell Basics



$$I = I_0 (e^{qV/kT} - 1) - I_{sc}$$

Light Absorption



Light intensity (x) $\propto e^{-\alpha x}$

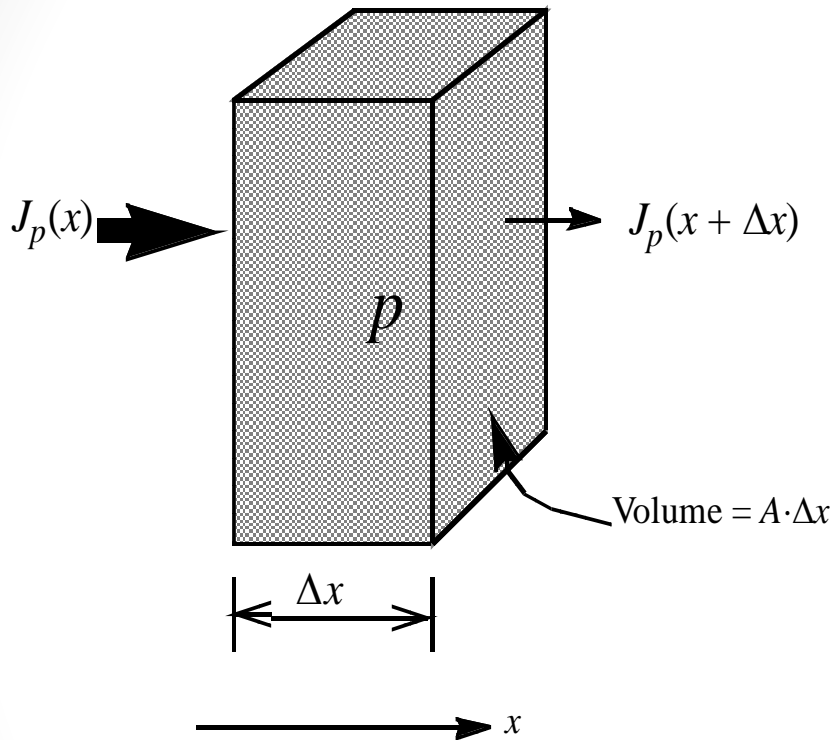
α (1/cm): absorption coefficient

$1/\alpha$: light penetration depth

$$\begin{aligned} \text{Photon Energy (eV)} &= \frac{hc}{\lambda} \\ &= \frac{1.24}{\lambda} (\mu\text{m}) \end{aligned}$$

A thinner layer of direct-gap semiconductor can absorb most of solar radiation than indirect-gap semiconductor.

Short-Circuit Current and Open-Circuit Voltage



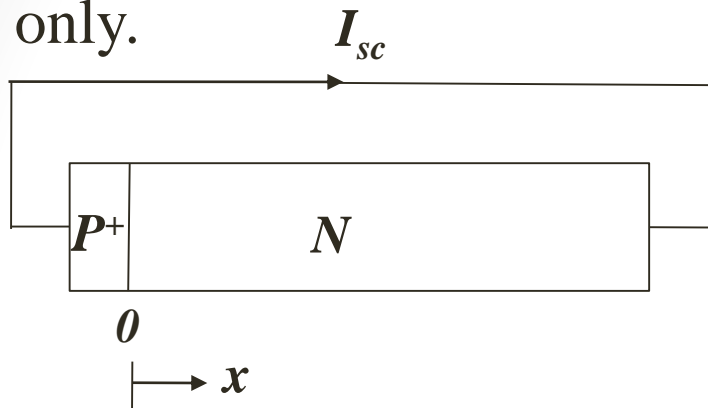
If light shines on the N-type semiconductor and generates holes (and electrons) at the rate of $G \text{ s}^{-1} \text{ cm}^{-3}$,

$$\frac{d^2 p'}{dx^2} = \frac{p'}{L_p^2} - \frac{G}{D_p}$$

If the sample is uniform (no PN junction),
 $d^2 p' / dx^2 = 0 \rightarrow p' = GL_p^2 / D_p = G\tau_p$

Solar Cell Short-Circuit Current, I_{sc}

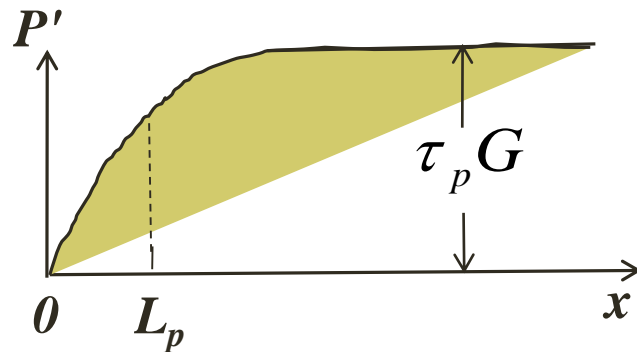
Assume very thin P+ layer and carrier generation in N region only.



$$p'(\infty) = L_p^2 \frac{G}{D_p} = \tau_p G$$

$$p'(0) = 0$$

$$p'(x) = \tau_p G (1 - e^{-x/L_p})$$



$$J_p = -qD_p \frac{dp'(x)}{dx} = q \frac{D_p}{L_p} \tau_p G e^{-x/L_p}$$

$$I_{sc} = A J_p(0) = A q L_p G$$

G is really not uniform. L_p needs to be larger than the light penetration depth to collect most of the generated carriers.

Open-Circuit Voltage

- Total current is I_{SC} plus the PV diode (dark) current:

$$I = Aq \frac{n_i^2}{N_d} \frac{D_p}{L_p} (e^{qV/kT} - 1) - AqL_p G$$

- Solve for the open-circuit voltage (V_{oc}) by setting $I=0$ (assuming $e^{qV_{oc}/kT} \gg 1$)

$$0 = \frac{n_i^2}{N_d} \frac{D_p}{L_p} e^{qV_{oc}/kT} - L_p G$$

$$V_{oc} = \frac{kT}{q} \ln(\tau_p G N_d / n_i^2)$$

How to raise V_{oc} ?

Output Power

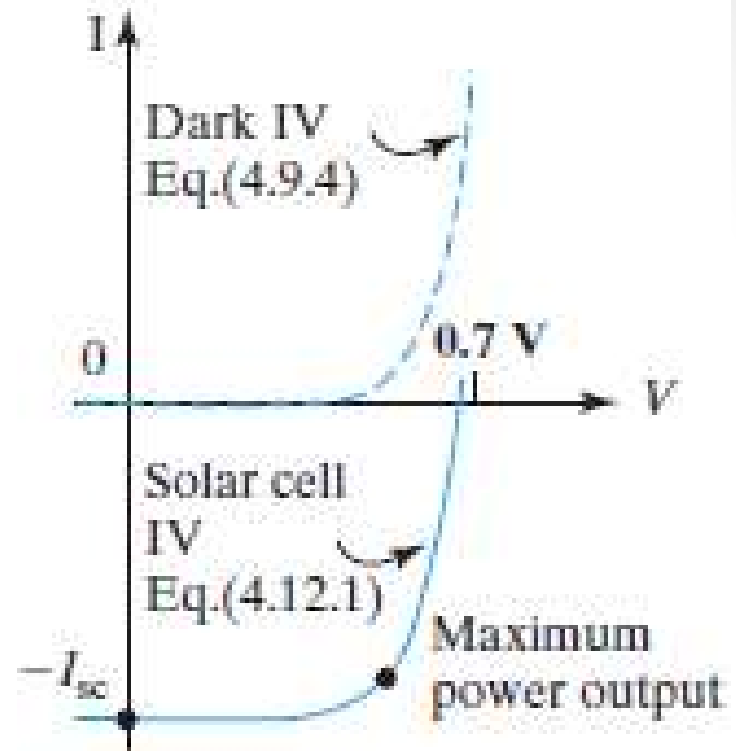
A particular operating point on the solar cell I-V curve maximizes the output power ($I \times V$).

$$\text{Output Power} = I_{sc} \times V_{oc} \times FF$$

- Si solar cell with 15-20% efficiency dominates the market now

- Theoretically, the highest efficiency (~24%) can be obtained with $1.9\text{eV} > E_g > 1.2\text{eV}$. Larger E_g lead to too low I_{sc} (low light absorption); smaller E_g leads to too low V_{oc} .

- **Tandem solar cells** gets 35% efficiency using large *and* small E_g materials tailored to the short and long wavelength solar light.



Solar cell

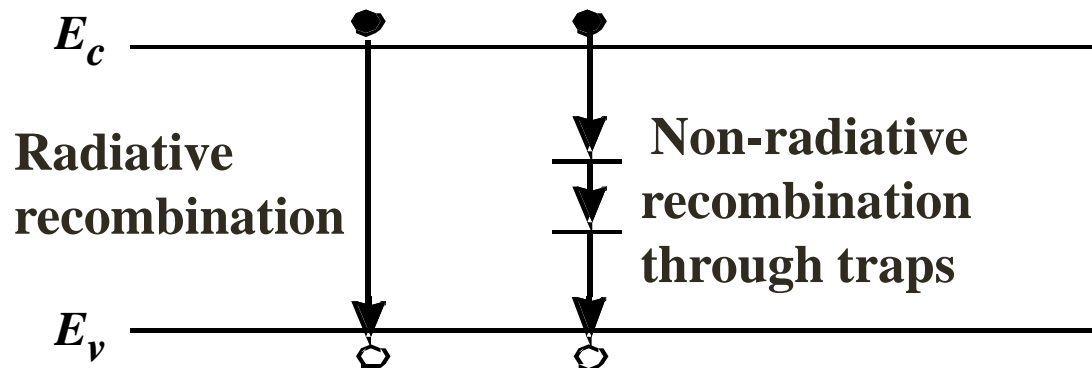
$$\text{Solar cell power} = I_{sc} \times V_{oc} \times FF$$

- ~100 μm Si or <1 μm direct-gap semiconductor can absorb most of solar photons with energy larger than E_g .
- Carriers generated within diffusion length from the junction can be collected and contribute to the Short Circuit Current I_{sc} .
- Theoretically, the highest efficiency (~24%) can be obtained with 1.9eV $>E_g>1.2\text{eV}$. Larger E_g lead to too low I_{sc} (low light absorption); smaller E_g leads to too low Open Circuit Voltage V_{oc} .
- Si cells with ~15% efficiency dominate the market. >2x cost reduction (including package and installation) is required to achieve cost parity with base-load non-renewable electricity.

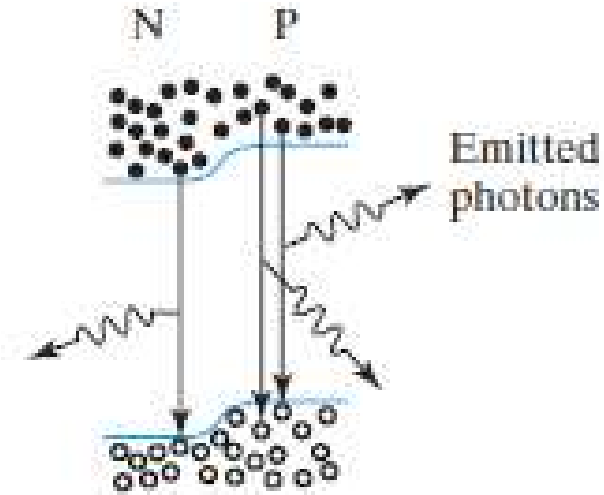
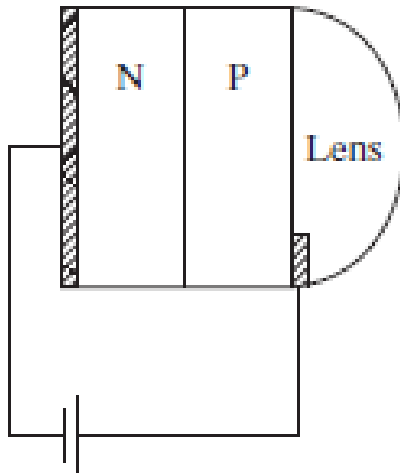
Light Emitting Diodes and Solid-State Lighting

Light emitting diodes (LEDs)

- LEDs are made of compound semiconductors such as InP and GaN.
- Light is emitted when electron and hole undergo *radiative recombination*.



LED Materials and Structure



$$\text{LED wavelength } (\mu\text{ m}) = \frac{1.24}{\text{photon energy}} \approx \frac{1.24}{E_g \text{ (eV)}}$$

LED Materials and Structure

	E_g (eV)	Wavelength (μm)	Color	Lattice constant (\AA)	
InAs	0.36	3.44		6.05	
InN	0.65	1.91		3.45	
InP	1.36	0.92		5.87	
GaAs	1.42	0.87		5.66	
GaP	2.26	0.55		5.46	
AlP	3.39	0.51		5.45	
GaN	2.45	0.37		3.19	
AlN	6.20	0.20		UV	3.11

Light-emitting diode materials

compound semiconductors

binary semiconductors:

- Ex: GaAs, efficient emitter

ternary semiconductor :

- Ex: $\text{GaAs}_{1-x}\text{P}_x$, tunable E_g (to vary the color)

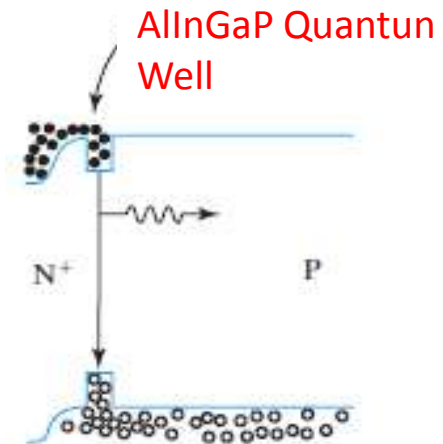
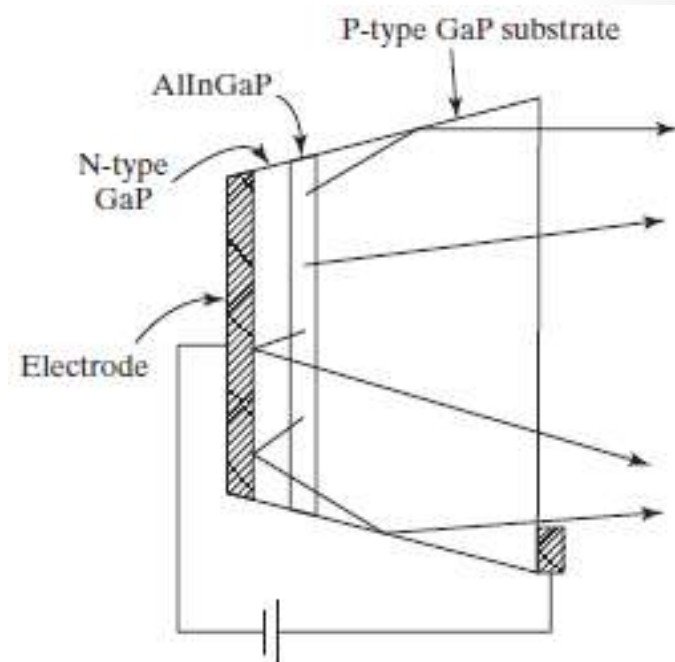
quaternary

semiconductors:

- Ex: AlInGaP, tunable E_g and lattice constant (for growing high quality epitaxial films on inexpensive substrates)

Common LEDs

Spectral range	Material System	Substrate	Example Applications
Infrared	InGaAsP	InP	Optical communication
Infrared -Red	GaAsP	GaAs	Indicator lamps. Remote control
Red-Yellow	AlInGaP	GaA or GaP	Optical communication. High-brightness traffic signal lights
Green-Blue	InGaN	GaN or sapphire	High brightness signal lights. Video billboards
Blue-UV	AlInGaN	GaN or sapphire	Solid-state lighting
Red-Blue	Organic semiconductors	glass	Displays



Solid-State Lighting

luminosity (lumen, lm): a measure of visible light energy normalized to the sensitivity of the human eye at different wavelengths

Incandescent lamp	Compact fluorescent lamp	Tube fluorescent lamp	White LED	Theoretical limit at peak of eye sensitivity ($\lambda=555\text{nm}$)	Theoretical limit (white light)
17	60	50-100	90-?	683	~340

Luminous efficacy of lamps in lumen/watt

Organic Light Emitting Diodes (OLED) :

has lower efficacy than nitride or aluminide based compound semiconductor LEDs.

Terms: **luminosity** measured in **lumens**. **luminous efficacy**

LED and Solid-State Lighting

- Electron-hole recombination in direct-gap semiconductors such as GaAs produce light.
- Ternary semiconductors such as GaAsP provide tunable E_g and LED color.
- Quaternary semiconductors such as AlInGaP provide tunable E_g and lattice constants for high quality epitaxial growth on inexpensive substrates.
- Beyond displays, communication, and traffic lights, a new application is space lighting with luminous efficacy $>5x$ higher than incandescent lamps. White light can be obtained with UV LED and phosphors. Cost still an issue.
- Organic semiconductor is an important low-cost LED material class.

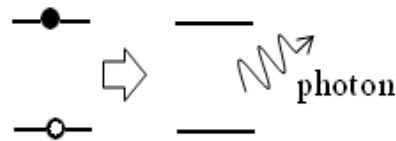
Diode Lasers

Light Amplification

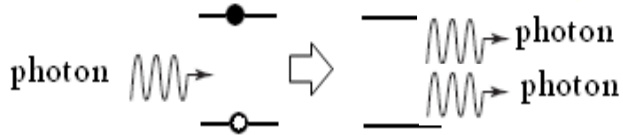
(a) Absorption



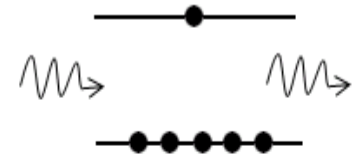
(b) Spontaneous Emission



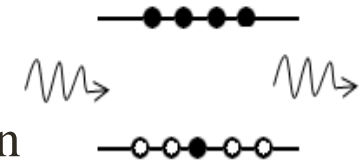
(c) Stimulated Emission



(d) Net Light Absorption



(e) Net Light Amplification



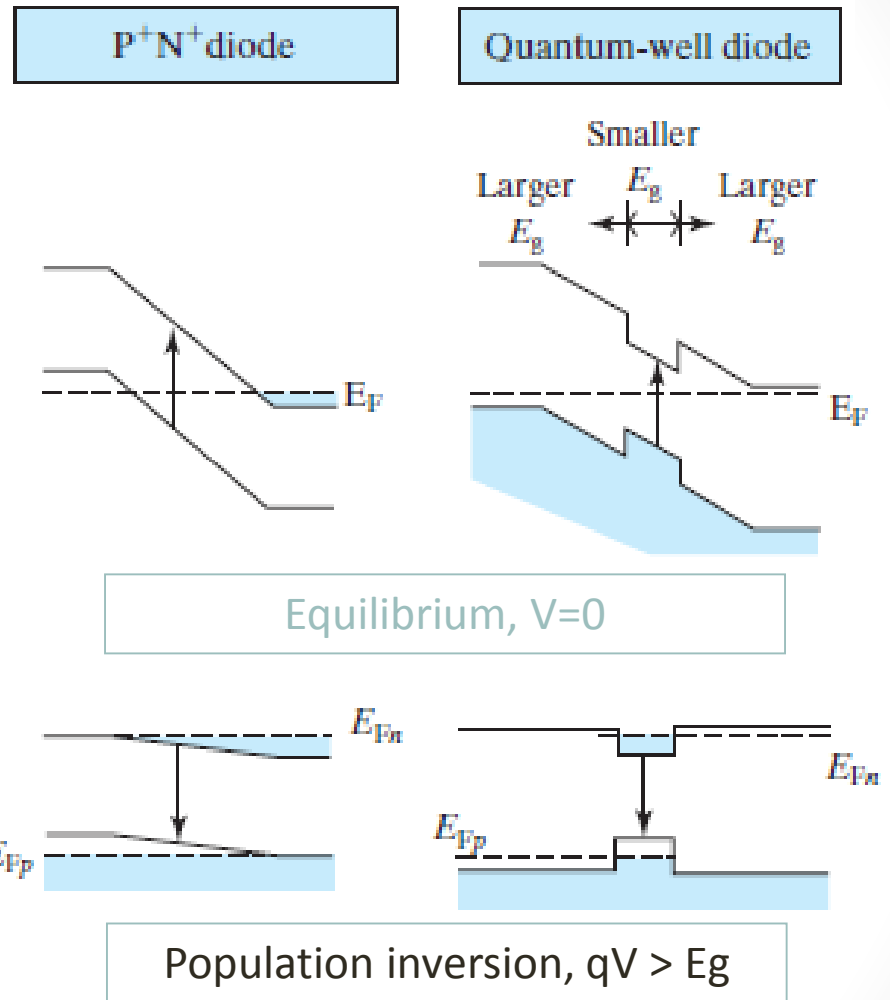
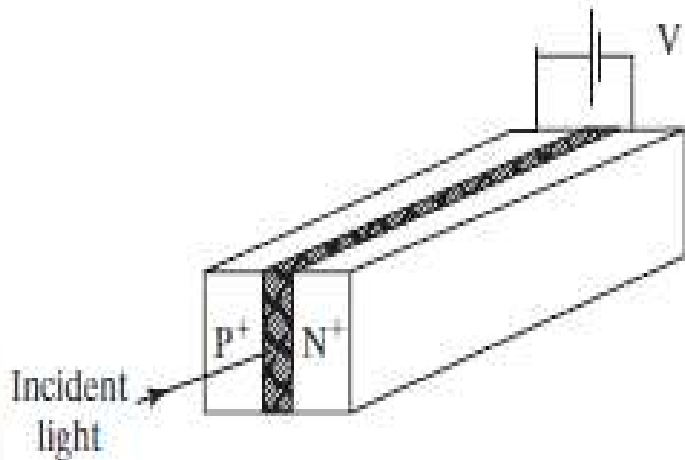
Light amplification requires population inversion: electron occupation probability is larger for higher E states than lower E states.

Stimulated emission: emitted photon has identical frequency and directionality as the stimulating photon; light wave is amplified.

Light Amplification in PN Diode

Population inversion is achieved when

$$qV = E_{fn} - E_{fp} > E_g$$

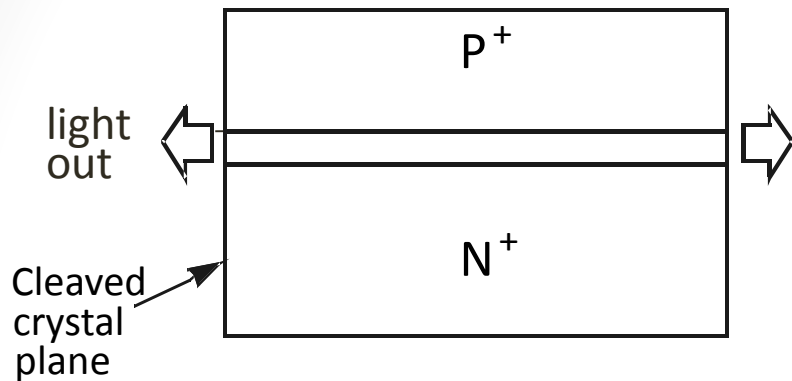


Optical Feedback and Laser

Laser threshold is reached (light intensity grows by feedback) when

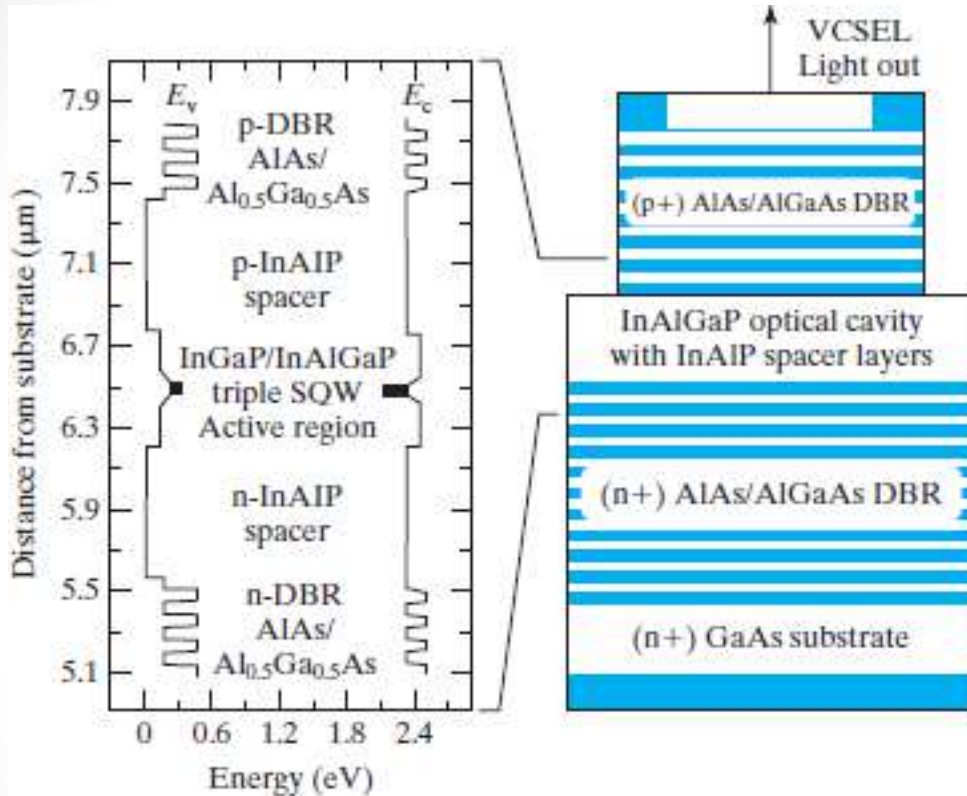
$$R_1 \times R_2 \times G \geq 1$$

- **R1, R2**: reflectivities of the two ends
- **G** : light amplification factor (gain) for a round-trip travel of the light through the diode



Light intensity grows until $R_1 \times R_2 \times G = 1$, when the light intensity is just large enough to stimulate carrier recombinations at the same rate the carriers are injected by the diode current.

Optical Feedback and Laser Diode



- *Distributed Bragg reflector (DBR)* reflects light with multi-layers of semiconductors.
- *Vertical-cavity surface-emitting laser (VCSEL)* is shown on the left.

Quantum-well laser has smaller threshold current because fewer carriers are needed to achieve population inversion in the small volume of the thin small- E_g well.

Laser Applications

Red diode lasers: CD, DVD reader/writer

Blue diode lasers: Blu-ray DVD (higher storage density)

1.55 μm infrared diode lasers: Fiber-optic communication

Photodiodes

Photodiodes: Reverse biased PN diode. Detects photo-generated current (similar to I_{sc} of solar cell) for optical communication, DVD reader, etc.

Avalanche photodiodes: Photodiodes operating near avalanche breakdown amplifies photocurrent by impact ionization.

Laser Diodes

- Light is amplified under the condition of population inversion – states at higher E have higher probability of occupation than states at lower E .
- Population inversion occurs when diode forward bias $qV > E_g$.
- Optical feedback is provided with cleaved surfaces or distributed Bragg reflectors.
- When the round-trip gain (including loss at reflector) exceeds unity, laser threshold is reached.
- Quantum-well structures significantly reduce the threshold currents.
- Purity of laser light frequency enables long-distance fiber-optic communication. Purity of light direction allows focusing to tiny spots and enables DVD writer/reader and other application.

Numericals

1. In an intrinsic semiconductor with $E_g = 0.7$ eV, Determine the position of Fermi level at $T=300$ K if $m_p^* = 6m_e^*$. Also Calculate the density of holes and electrons at 300 K. How do these quantities alter if $E_g = 7$ eV.
2. In a pure semiconductor, the number of electrons is 6×10^{19} /m³. How many holes are there on a sample of size 1 cm x 1cm x 1mm?

3. The band gap for Si is 1.1 eV. (a) Find the ratio of band gap to kT for silicon at room temperature 300 K. (b) At what temperature does this ratio becomes one tenth of the value at 300 K.

4. The concentration of conduction electrons in germanium is $6 \times 10^{19} / \text{m}^3$. When some phosphorus impurity is doped into germanium sample, the concentration of electron increases to $2 \times 10^{23} / \text{m}^3$. Find the concentration of holes in doped germanium.

5. In a p-n junction, the depletion region is 400nm wide and an electric field of 5×10^5 V/m exists in it. (a) Find the height of potential barrier.(b) What should be the minimum K.E. of a conduction electron which can diffuse from n-side to p-side?

6. The effective mass of hole and electron in GaAs are respectively 0.48 and 0.067 times the free electron mass. The band gap energy is 1.43 eV. How much above is its fermi-level from the top of the valence band at 300 K?

8. Pure silicon at 300K has electron and hole density each equal to $1.5 \times 10^{16} \text{ m}^{-3}$. One of every 1.0×10^7 atoms is replaced by a phosphorous atom.

- (a) What charge carrier density will the phosphorous add? Assume that all the donor electrons are in the conduction band.
- (b) Find the ratio of the charge carrier density in the doped silicon to that for the pure silicon.

Given: density of silicon = 2330 kg m^{-3} ; Molar mass of silicon = 28.1 g/mol ; Avogadro constant $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$.

9. The effective mass of the conduction electron in Si is 0.31 times the free electron mass. Find the conduction electron density at 300 K, assuming that the Fermi level lies exactly at the center of the energy band gap (= 1.11 eV).

10. In intrinsic GaAs, the effective masses of electron and hole respectively are 0.068 and 0.50 times the electron mass. The energy band gap is 1.43 eV. Determine the carrier density at 300K.