Engineering Drawing Principles L-5

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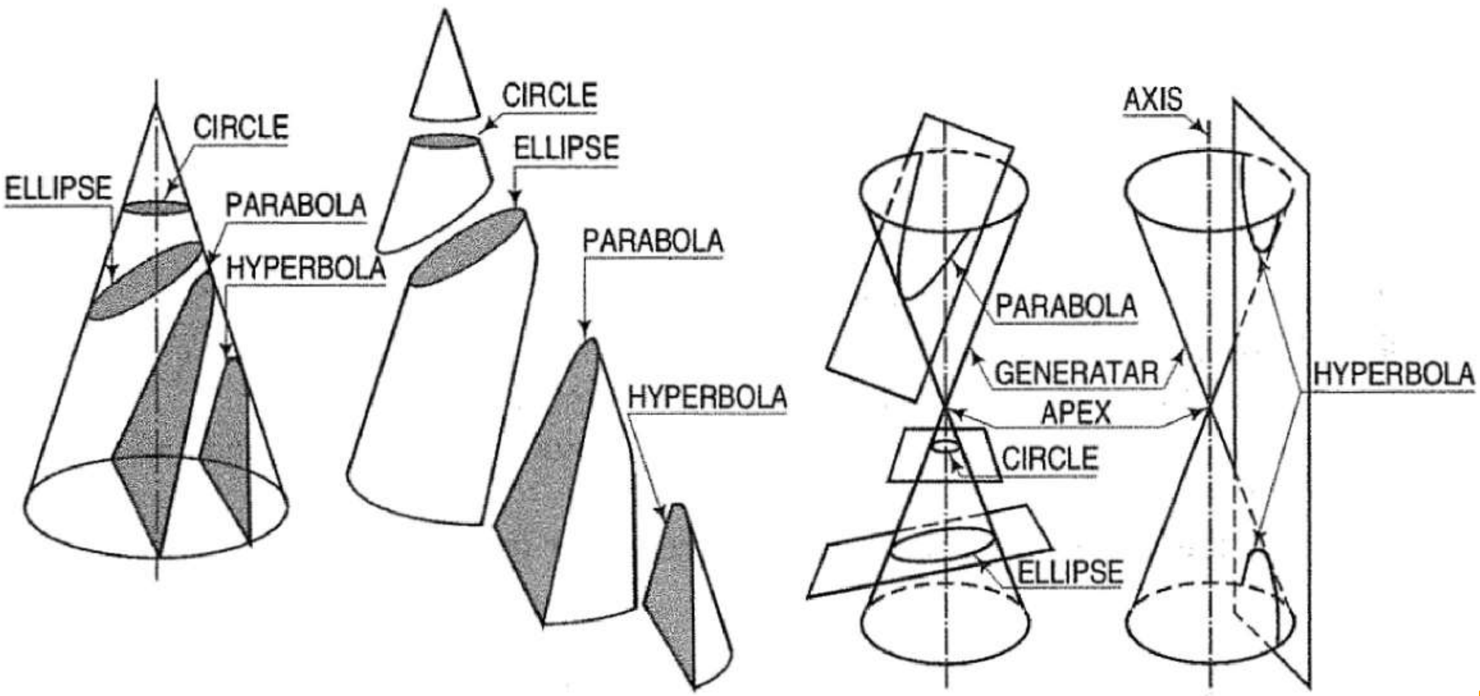
Curves used in Engineering Drawings

The profile of number of objects consists of various types of curves which are commonly used in engineering practice as shown below:

1. Conic sections
2. Cycloidal curves
3. Involute
4. Evolutes
5. Spirals
6. Helix

Conic sections

The sections obtained by the intersections of a right circular cone by a plane in different positions relative to the axis of cone are called conics.



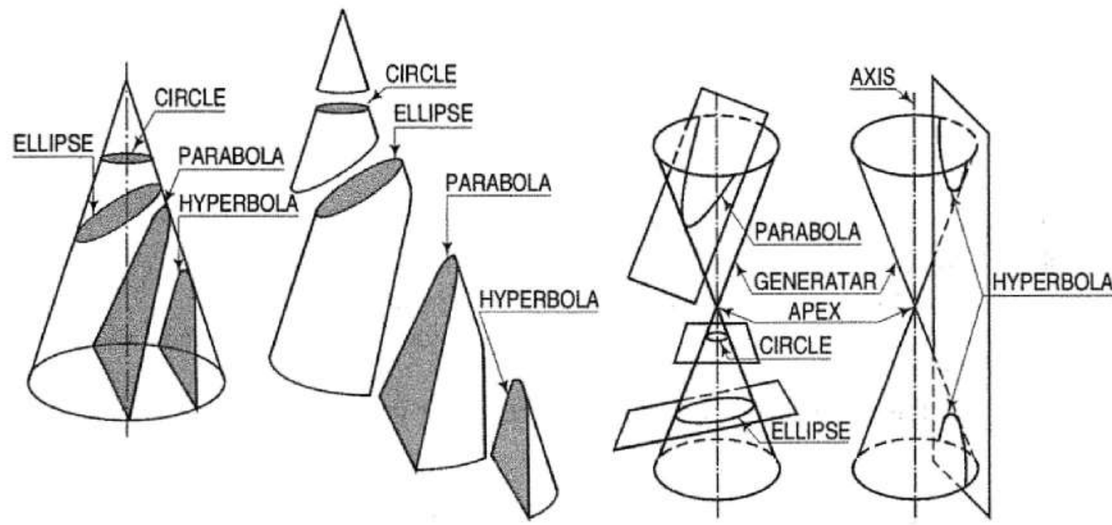
Conic sections

Conic sections

i. When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse.*

ii. When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola*.

iii. A *hyperbola* is a plane curve having two separate parts or branches, formed when two cones that point towards one another are intersected by a plane that is parallel to the axes of the cones.



Conic sections

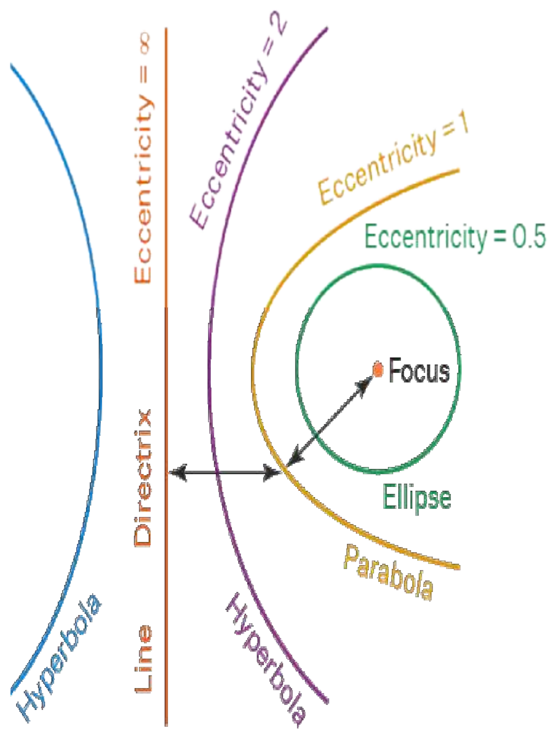
Conic sections

The conic may be defined as the locus of a point moving in a plane in such a way that the ratio of its distances from a fixed point and a fixed straight line is always constant. The fixed point is called the *focus* and the fixed line, the *directrix.*

The ratio of distance of the point from the focus/distance of the point from the directrix is called *eccentricity* and is denoted by *e.* It is always less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola i.e.

(i) ellipse : $e < 1$
(ii) parabola : $e = 1$
(iii) hyperbola : $e > 1$.

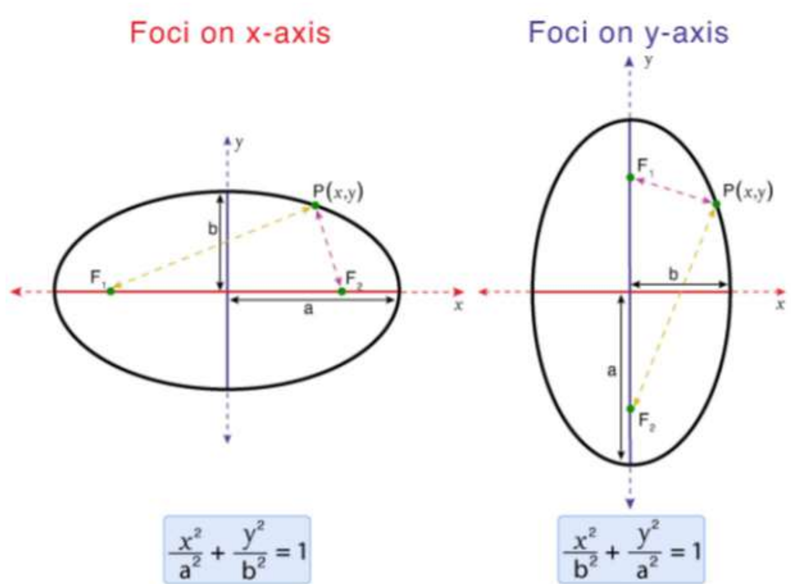
The line passing through the focus and perpendicular to the directrix is called the *axis.* The point at which the conic cuts its axis is called the *vertex.*

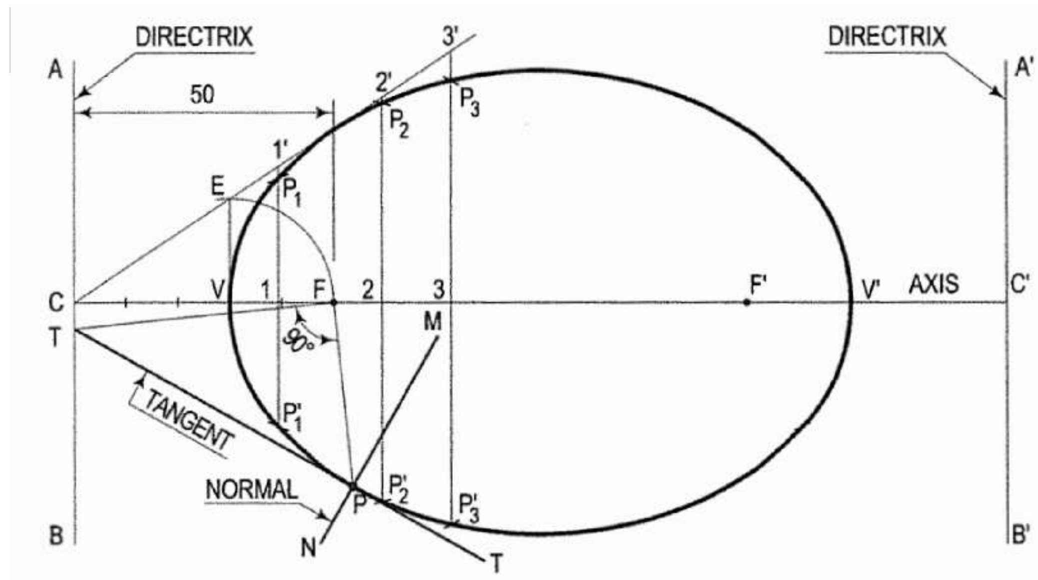


Conic Sections

Ellipse

Use of elliptical curves is made in arches, bridges, dams, monuments, manholes, glands and stuffing-boxes etc. Mathematically an ellipse can be described by



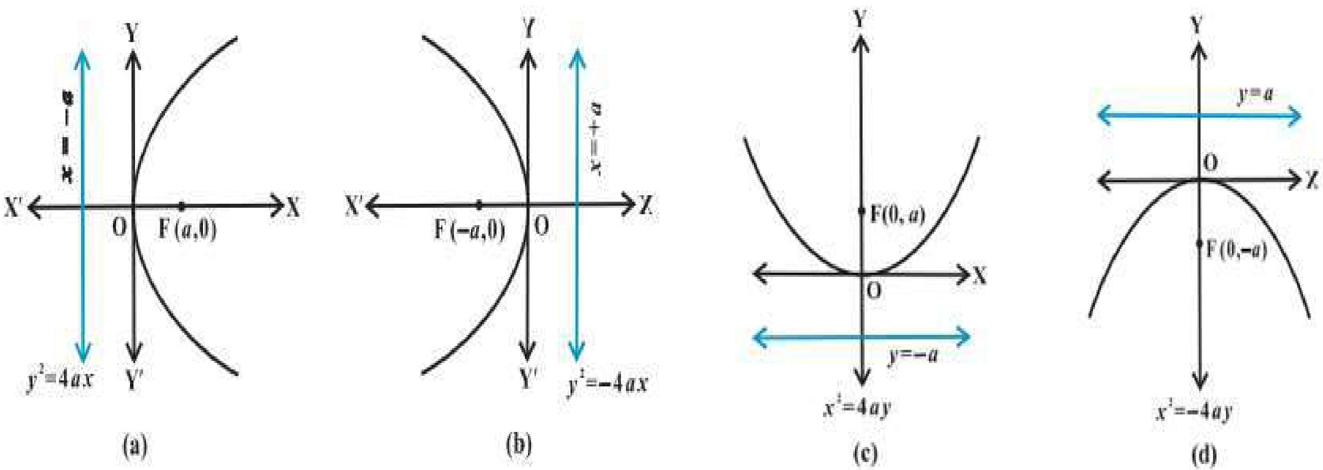


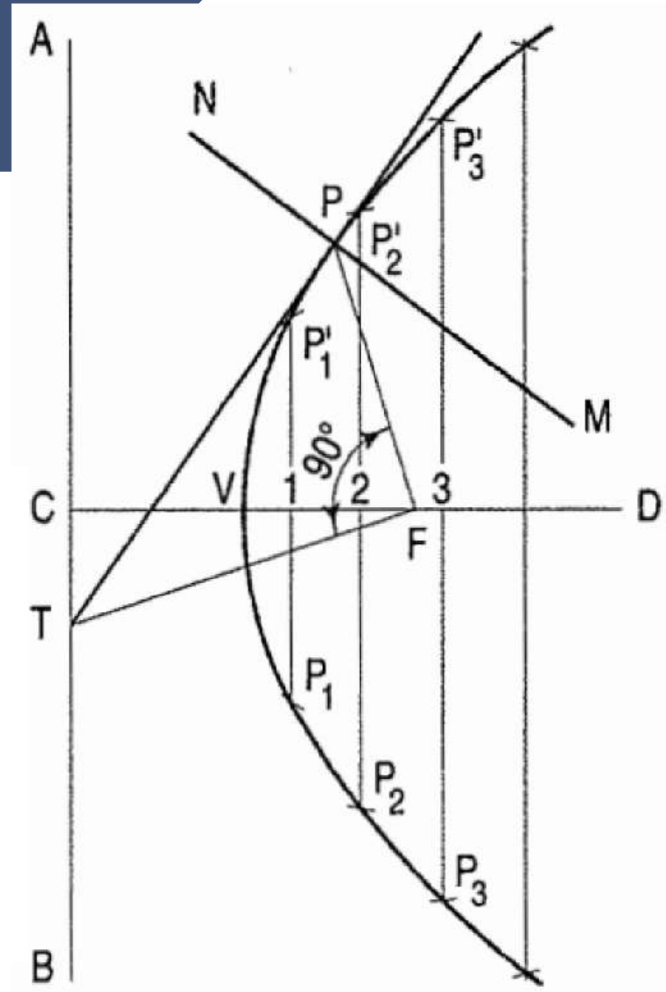
Directrix and focus

Conic Sections

Parabola

Use of parabolic curves is made in arches, bridges, sound reflectors, light reflectors etc. Mathematically a parabola can be described by an equation $y^2 = 4ax$ or $x^2 = 4ay$



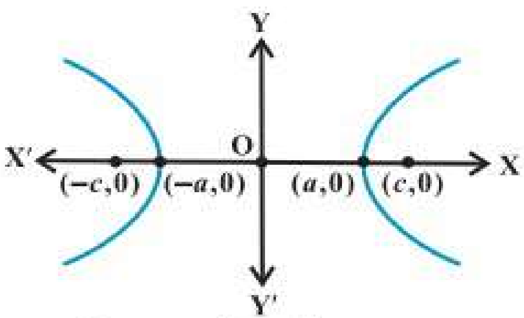


Directrix and focus

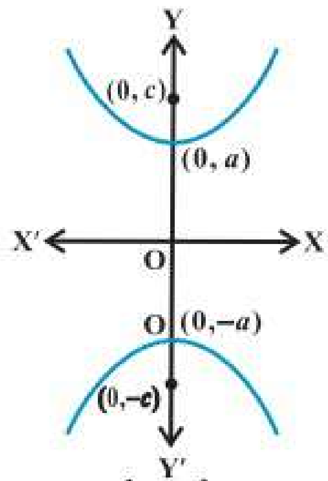
Hyperbola

Use of hyperbolical curves is made in cooling towers, water channels etc.

Rectangular hyperbola: It is a curve traced out by a point moving in such a way that the product of its distances from two fixed lines at right angles to each other is a constant. The fixed lines are called *asymptotes.*



(a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



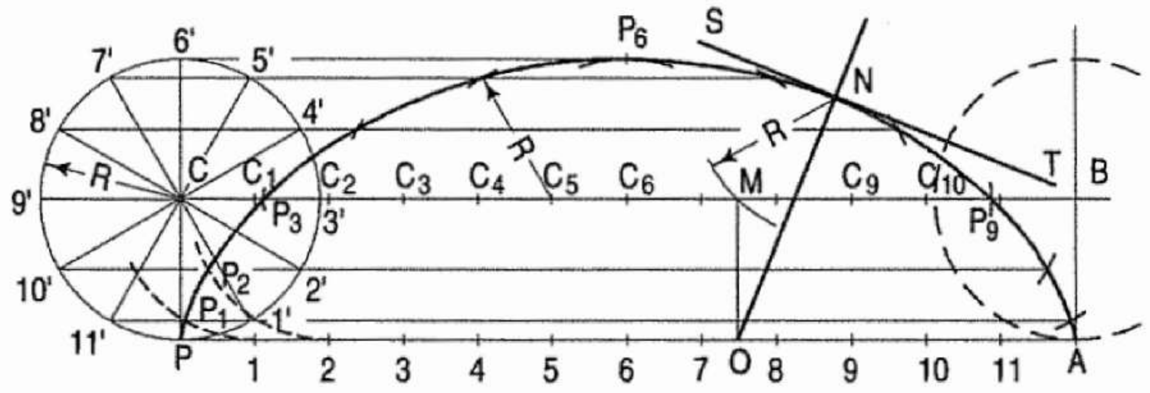
(b) $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Cycloidal Curves

These curves are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line or a circle. The rolling circle is called *generating circle* and the fixed straight line or circle is termed *directing line* or *directing circle.* Cycloidal curves are used in tooth profile of gears of a dial gauge.

Cycloid is a curve generated by a point on the circumference of a circle which rolls along a straight line. It can be described by an equation,

$y = a\,(1 - \cos\Phi)$ or $x = a\,(\Phi - \sin\Phi)$

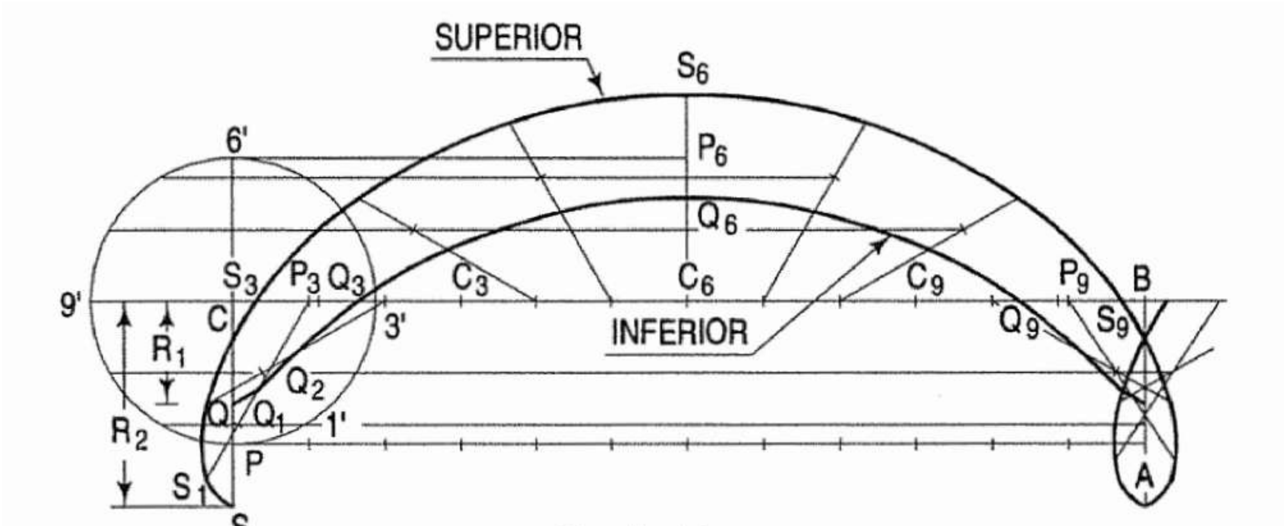


Cycloid

Trochoid

Trochoid is a curve generated by a point fixed to a circle, within or outside its circumference, as the circle rolls along a straight line.

When the point is within the circle, the curve is called *an inferior trochoid* and when outside the circle, it is termed a *superior trochoid*



Trochoids

Epicycloid

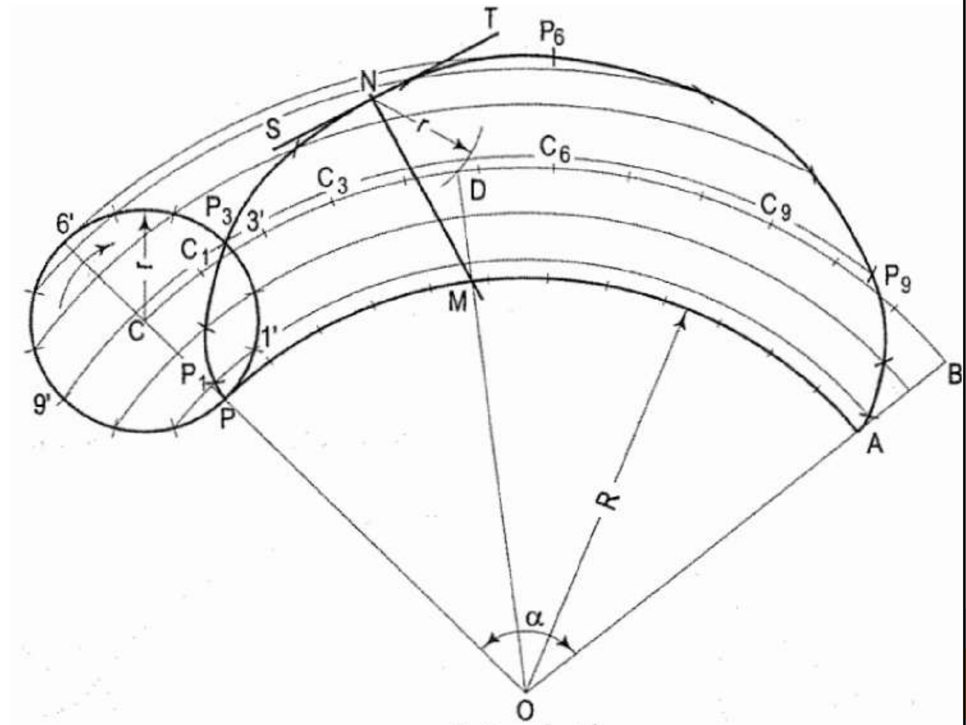
The curve generated by a point on the circumference of a circle, which rolls without slipping along another circle *outside* it, is called an *epicycloid. The epicycloid can be represented mathematically by*

$$x = (a + b)\cos\theta - a\cos\left(\frac{a+b}{a}\theta\right),$$

$$y = (a + b)\sin\theta - a\sin\left(\frac{a+b}{a}\theta\right)$$

where a is the radius of rolling circle.

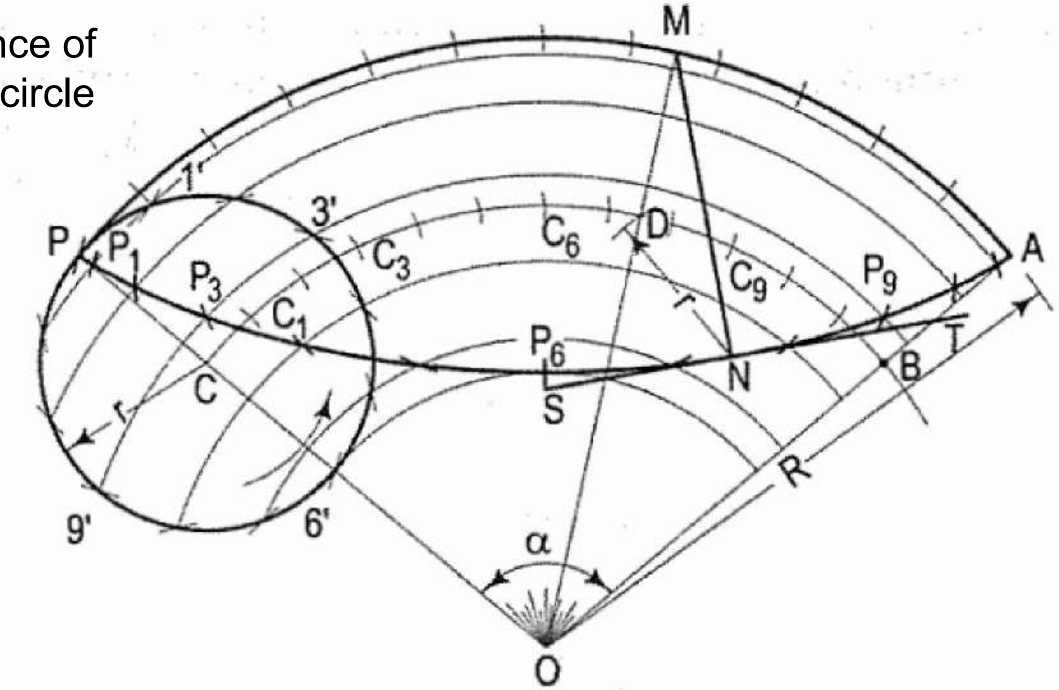
When the circle rolls along another circle *inside* it, the curve is called a *hypocycloid.*
It can be represented by mathematically $x = a\cos^3\theta$, $y = a\sin^3\theta$.



Epicycloid

Hypocycloid

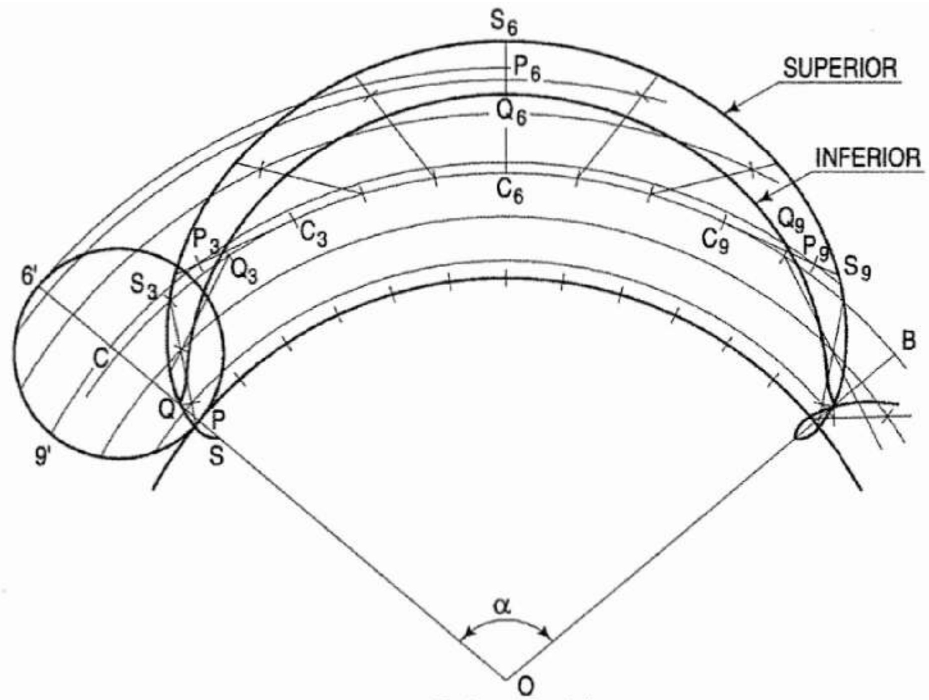
The curve generated by a point on the circumference of a circle, which rolls without slipping along another circle *inside* it, is called an *hypocycloid*



Hypocycloid

Epitrochoid

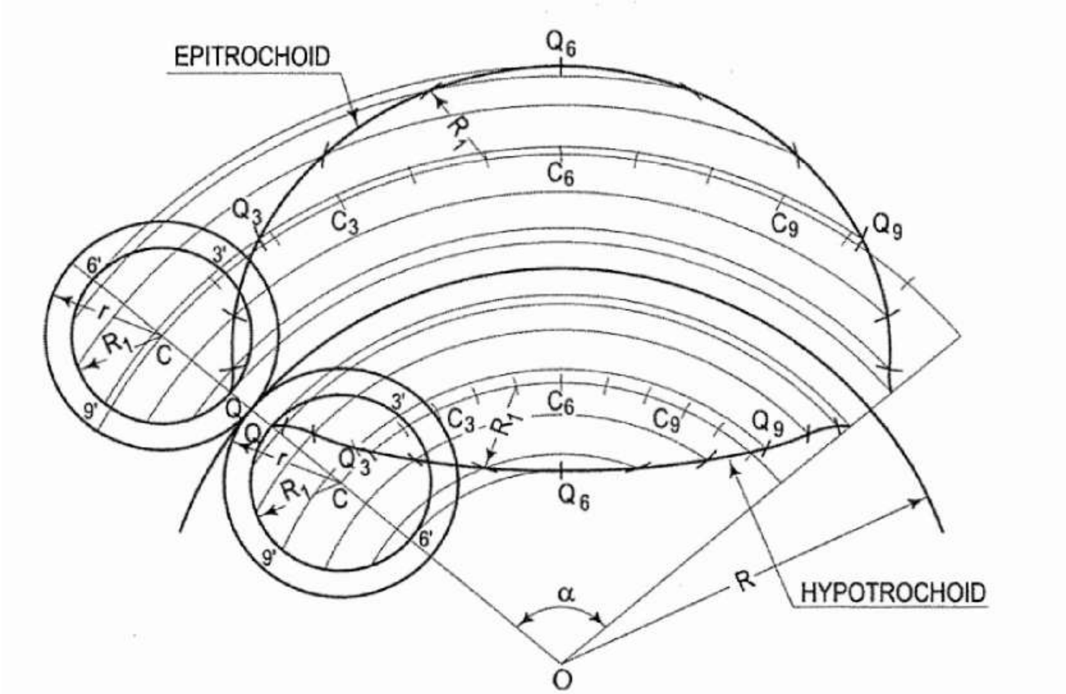
Epitrochoid is a curve generated by a point fixed to a circle (within or outside its circumference, but in the same plane) rolling on the outside.



Epitrochoids

Hypotrochoid

When the circle rolls inside another circle, the curve is called a hypotrochoid. The curve is termed inferior or superior, according to the position of the point being inside or outside the rolling circle

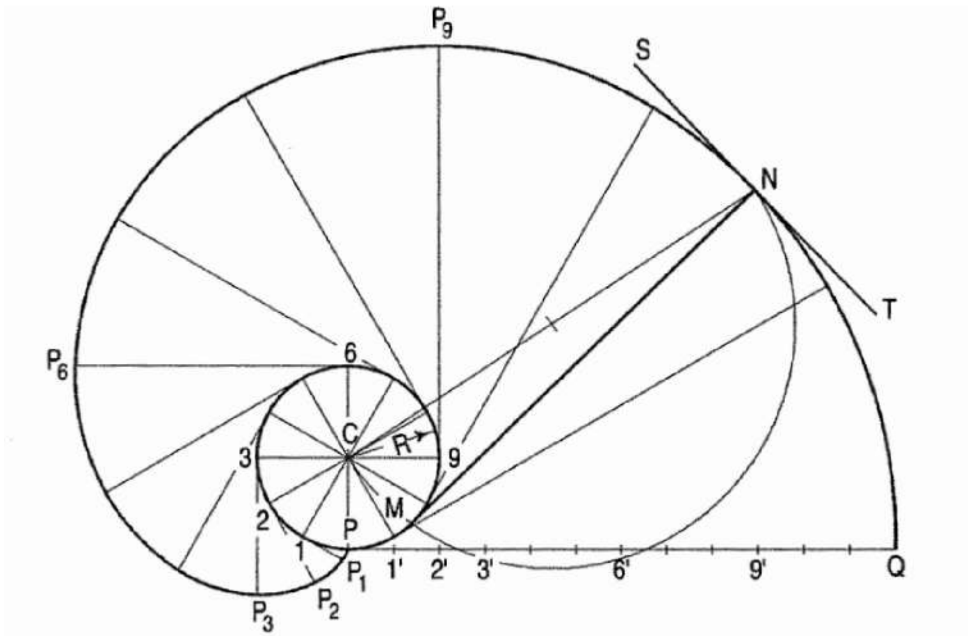


Inferior epitrochoid and hypotrochoid

Involute

The involute is a curve traced out by an end of a piece of thread unwound from a circle or a polygon, the thread being kept tight. It may also be defined as a curve traced out by a point in a straight line which rolls without slipping along a circle or a polygon. Involute of a circle is used as teeth profile of gear wheel.

Mathematically it can be described by
x = r cosθ + rθsinθ, y = rsinθ - rθcosθ,
where "r" is the radius of a circle



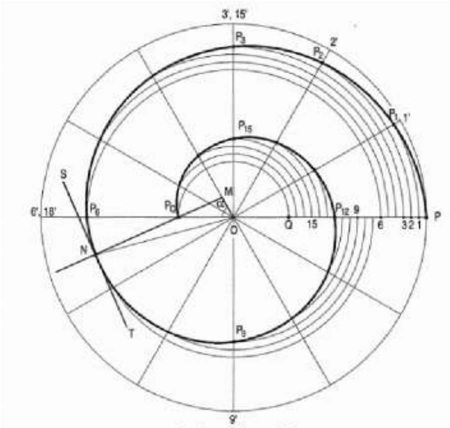
Involute

Spirals

If a line rotates in a plane about one of its ends and if at the same time, a point moves along the line continuously in one direction, the curve traced out by the moving point is called a *spiral.* The point about which the line rotates is called a *pole.*

The line joining any point on the curve with the pole is called the *radius vector*
The angle between this line and the line in its initial position is called the *vectorial angle.*

Each complete revolution of the curve is termed the *convolution.* A spiral may make any number of convolutions before reaching the pole.

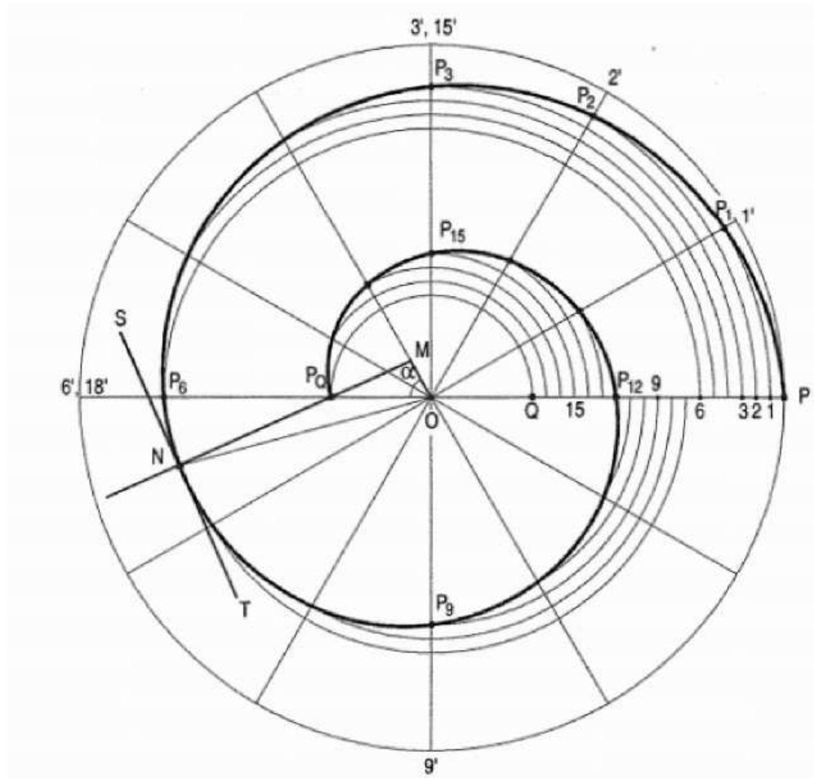


Archemedian spiral

Archemedian Spirals

It is a curve traced out by a point moving in such a way that its movement towards or away from the pole is uniform with the increase of the vectorial angle from the starting line.

The use of this curve is made in teeth profiles of helical gears, profiles of cam

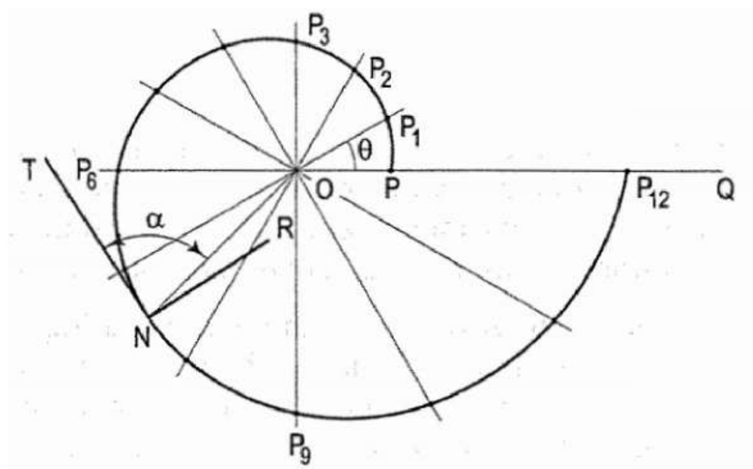


Archemedian spiral

Archemedian Spirals

In a *logarithmic spiral,* the ratio of the lengths of consecutive radius vectors enclosing equal angles is always constant. In other words the values of vectorial *angles* are in *arithmetical progression* and the corresponding values of *radius* vectors are in *geometrical progression.*

The logarithmic spiral is also known as *equiangular spiral* because of its property that the angle which the tangent at any point on the curve makes with the radius vector at that point is constant



Helix

Helix is defined as a curve, generated by a point, moving around the surface of a right circular cylinder or a right circular cone in such a way that, its axial advance, i.e. its movement in the direction of the axis of the cylinder or the cone is uniform with its movement around the surface of the cylinder or the cone.