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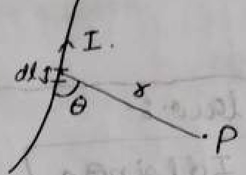
{Magnetism}

UNIT-2

• **Biot-Savart law:**

$$\boxed{dB = \frac{\mu_0}{4\pi} \times \frac{i(dl \times r)}{r^3}}$$



$dB \propto I$

$\propto dl$

$\propto 1/r^2$

$\propto \sin\theta$

$|\vec{dB}| \propto \frac{I\,dl \sin\theta}{r^2}$

$$\boxed{|\vec{dB}| = \frac{\mu_0}{4\pi} \times \frac{I\,dl\sin\theta}{r^2}}$$

$\therefore \mu_0 = \frac{|\vec{dB}|\, 4\pi r^2}{I\,dl\sin\theta} = \frac{Tm^2}{Am} \Rightarrow TmA^{-1}$

μ_0 units

$$\boxed{\frac{\mu_0}{4\pi} = 10^{-7}\ TmA^{-1}}$$

absolute permeability of free space

$$\boxed{\therefore dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl\sin\theta}{r^2}\,\hat{n}} \Rightarrow \boxed{\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\vec{dl} \times \vec{r}}{r^3}}$$

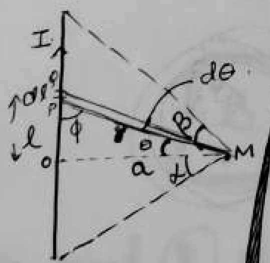
vector form

$$\hat{n} = \frac{I\vec{dl} \times \vec{r}}{I\,dl\sin\theta\, r}$$

$\frac{\vec{n}}{|n|}$

• **Application**

↳ MF due to a st. current carrying wire.



$\to |\vec{dB}| = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl\sin\phi}{r^2}$

$\theta + \phi = 90°$

$\phi = 90° - \theta$

$\sin\phi = \cos\theta$.

$\boxed{\frac{l}{a} = \tan\theta}$

$l = a\tan\theta$

$dl = a\sec^2\theta\, d\theta$

$\frac{a}{r} = \cos\theta$

$r = \frac{a}{\cos\theta} \Rightarrow a\sec\theta$

$\therefore \vec{dB} = \frac{\mu_0}{4\pi} \frac{I\cos\theta\, d\theta}{a}$.

$= \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \int_{\alpha}^{\beta} \cos\theta\, d\theta$

$$\boxed{|\vec{B}| = \frac{\mu_0}{4\pi} \frac{I}{a} (\sin\beta - \sin\alpha)}.$$

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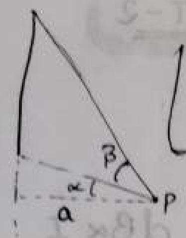
cases:



$$\boxed{B = \frac{\mu_0 I}{4\pi a}\left(\sin\beta + \sin\alpha\right)}$$

$\alpha, \beta = 90°$

$$B = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$

Biot-savart law:-

$$\underbrace{\boxed{dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}\hat{n}}}_{\text{normal form.}} \quad \text{or} \quad \underbrace{\boxed{dB = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}}}_{\text{vector form}}$$

- $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$
  $\nabla \times E = 0$

	EP.	EF.
Monop.	$1/r$	$1/r^2$
Dipole	$1/r^2$	$1/r^3$
Quadr.	$1/r^3$	$1/r^4$
Octa.	$1/r^4$	$1/r^5$.

- EF due to dipole =
- EP due to dipole =

Lapalace : $\nabla^2 V = -\rho/\varepsilon_0$

Poison : $\nabla^2 V = 0$

when $\rho \neq 0$ Poison
when $\rho = 0$ Lapalace.

uniquness theoram $\nabla^2 V = 0$

$\rho = 0$

Boundary condi.

① EF : $E_{\parallel}^a - E_{\parallel}^b = 0$
$E_{\perp}^a - E_{\perp}^b = \sigma/\varepsilon_0$

$\varepsilon_0 E - \vec{P} = \vec{D}$

$\vec{P}$ = polarisation vector
$\vec{D}$ = Electric displ. vec.

$\vec{EP} \Rightarrow \vec{D}_{\parallel}^a - \vec{D}_{\parallel}^b = \vec{P}_{\parallel}^a - \vec{P}_{\parallel}^b$
$\vec{D}_{\perp}^a - \vec{D}_{\perp}^b = \sigma_f$

$\sigma_b = \vec{P} \cdot \hat{n}$

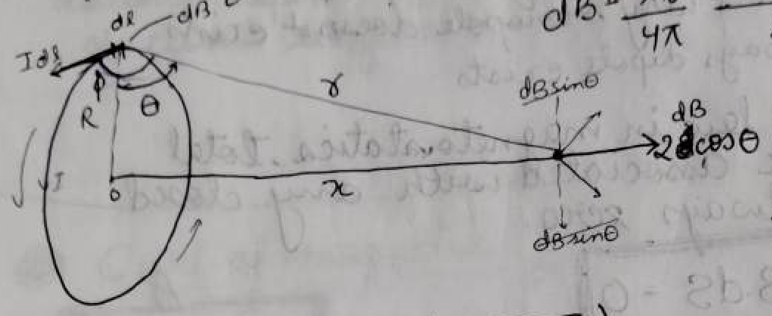
$\rho_b = -\nabla \cdot P$

Magnetic field due to circular current loop:-

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2}$$

$\hookrightarrow \phi = 90^\circ$

$$\therefore dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$



$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl}{(x^2+R^2)} \qquad \{ r = \sqrt{x^2+R^2} \}$$

$\cos\theta = \frac{R}{r}$

$$\hookrightarrow \therefore dB\cos\theta = \frac{\mu_0}{4\pi}\frac{I\,dl}{x^2+R^2}\cos\theta$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{x^2+R^2}\cdot\frac{R}{r}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{x^2+R^2}\frac{R}{\sqrt{x^2+R^2}}$$

$$\int dB\cos\theta = \frac{\mu_0}{4\pi}\frac{IR}{(x^2+R^2)^{3/2}}\int dl$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{(x^2+R^2)^{3/2}} \times 2\pi R$$

$$\boxed{B = \frac{\mu_0}{2}\frac{IR^2}{(x^2+R^2)^{3/2}}}$$

for 1 turn
multiply with n
$\hookrightarrow$ no. of turns.

If point (P) lies at the centre of loop $\rightarrow x = 0$

$\hookrightarrow B = \frac{\mu_0}{2}\frac{I}{R}$

- when $x >>> R \Rightarrow \frac{\mu_0}{2}\frac{IR^2}{x^3}$

Divergence of Magnetic field

$\boxed{\nabla \cdot B = 0}$ → that's why monopole doesn't exists
→ always dipole exists

according to Gauss law in magnetostatics, total magnetic flux associated with any closed surface is always zero.

$$\boxed{\phi_B = \oint B \cdot dS = 0}$$

By using divergence theorem.

$$\oint B \cdot ds = \int (\nabla \cdot B) d\tau$$

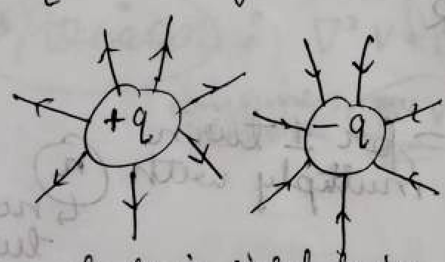
$$\int (\nabla \cdot B) d\tau = 0 .$$

$$\therefore \boxed{\nabla \cdot B = 0}$$

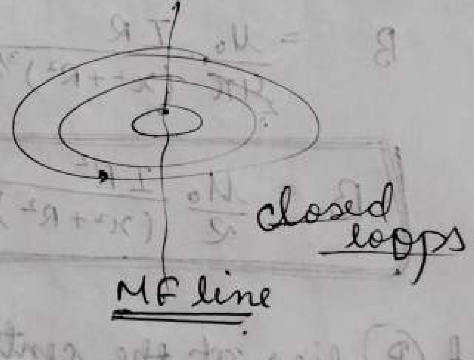
hence proved

⇒ Physical significance of $\boxed{\nabla \cdot B = 0}$

⟶ magnetic field lines neither have ending nor begining that is they are continous and follow right hand ~~rule~~ thumb rule.



electric field lines have ending and starting both



MF line

→ The no. of electric field lines entering any closed surface are equal to that lines coming out.

↳ ⇒ It means both the poles are equally strong.

→ The magnetic flux through a closed surface doesn't depend upon the size and location of closed surface.

Curl of magnetic field

$$\boxed{\nabla \times B = \mu_0 J}$$

→ acc. to Amphere circuital law the line integral of magnetic field induction B around a closed path in vaccum is equal to μ_0 times the total current passing through the closed path.

$$\Rightarrow \oint B \cdot dl = \mu_0 I$$

using strokes theoram

$$\int (\nabla \times B)\, dS = \mu_0 \int J\, dS$$

abhi bhi I ke equal hi he

$$\frac{I}{A} = J$$

$$\int (\nabla \times B)\, dS = \int \mu_0 J\, dS$$

$$\nabla \times B = \mu_0 J$$ hence proved

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Vector potential of a given magnetic field using Stoke's theoram

Introduct → Result

↳ magnetic vector potential ($\vec{A}$)
↳ potential due to magnetic field.

$\nabla \times B = \mu_0 J.$

$\nabla \cdot B = 0. \quad — (1)$

⇒ Div of curl of a vector is zero.

$\nabla \cdot (\text{curl } A) = 0.$

$\nabla \cdot (\nabla \times A) = 0. \quad — (11)$

on comp. (1) and (11).

$\boxed{B = \nabla \times A}$ — magnetic vector poten.

$\boxed{\nabla \times A = B}$ — no unique soln

↳ guage freedom.

we have freedom to choose A but such that it satisfies above eq. which is in case of a Solenoid.

that value of (A) will satisfies

$\nabla \cdot A = 0$

$\boxed{\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A} \quad — (3)$

$\nabla \times B = \mu_0 J.$
$\nabla \times (\nabla \times A) = \mu_0 J. \quad — (4).$

from (3) and (4).

$\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J.$

$-\nabla^2 A = \mu_0 J.$

$\boxed{\nabla^2 A = -\mu_0 J.}$

$\nabla^2(A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) = \mu_0 (J_x \hat{i} + J_y \hat{j} + \dots)$

$$\left.\begin{aligned} A_x &= -\mu_0 J_x \\ A_y &= -\mu_0 J_y \\ A_z &= -\mu_0 J_z. \end{aligned}\right\}$$

and $\nabla^2 A = -\mu_0 J.$ — Poisson eqn in magnetic field.

$\nabla^2 V = -\dfrac{\rho}{\varepsilon_0}$

On comparing thing with in EF

$V = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \dfrac{\rho}{r} d\tau$

similarly

$$\therefore \boxed{A_x = \frac{\mu_0}{4\pi} \int \frac{J_x}{r} d\tau}$$ → J is finite current density.

similarly we can use it in case of (σ, ρ, γ)

using stokes theoram

$$\phi_B = \int_s B\cdot ds$$

$$\phi_B = \int_s (\nabla\times A)\, ds.$$

$$\therefore \phi_B = \oint_l A\cdot dl.$$

$$d\phi_B = A\cdot dl.$$

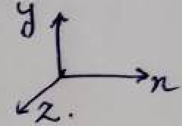
$$\boxed{A = \frac{d\phi_B}{dl}}$$

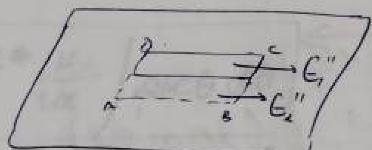
$E \Rightarrow$ $E_{\parallel}^{a} - E_{\parallel}^{b} = 0.$

$E_a^{\perp} - E_b^{\perp} = \sigma/\epsilon_0$

$D \Rightarrow$ $D_a^{\parallel} - D_b^{\parallel} = P_a^{\parallel} - P_b^{\parallel}$

$D_a^{\perp} - D_b^{\perp} = \sigma_f.$





proof $\oint E\cdot dl = 0.$

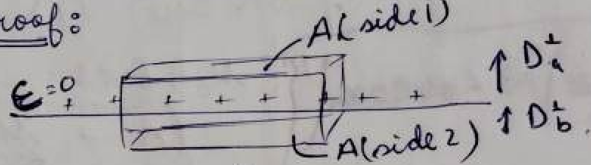
$\int E_1'' dl + \int E_2'' dl = 0$

$E_1'' dl - E_2'' dl = 0$

$\therefore E_1'' = E_2''$

$\therefore E_1'' - E_2'' = 0$

proof:



$$\int \nabla\cdot D\, d\tau = \int \rho_f\cdot d\tau$$

By using Diver. theor.

$$\oint D\cdot\hat{n}\, da = \int \frac{Q_{in}}{V}\, d\tau.$$

$$\oint D\cdot\hat{n}\, da = (\phi_{in})_{free}$$

$$\oint \vec{D}\cdot da = \int \rho_f\, d\tau.$$

$$\therefore \int_1 \vec{D}\cdot da + \int_2 \vec{D}\cdot da = \int \rho_f\, d\tau$$

$$\int \vec{D}_{\perp}^{a} da + \int \vec{D}_{\perp}^{b} da = \int \rho_f\, d\tau.$$

$$\int D_{\perp}^{a}(\hat{y})\, da(\hat{y}) + \int D_{\perp}^{b}(\hat{y})\, da(-\hat{y}) = \int \rho_f\, d\tau$$

$$\int D_{\perp}^{a} A - \int D_{\perp}^{b} A = \int \rho_f$$

$$\oint E\cdot da = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E\cdot da = \frac{\sigma A}{\varepsilon_0}$$

$$\int E_1'\cdot da + \int E_2\cdot da = \frac{\sigma A}{\varepsilon_0}$$

$$E_1 A - E_2 A = \frac{\sigma A}{\varepsilon_0}$$

$$(E_1 - E_2)\cancel{A} = \frac{\sigma}{\varepsilon_0}\cancel{A}$$

$$E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0}$$

$$\nabla\times D = \nabla\times P$$

$$\int(\nabla\times D)\,da = \int(\nabla\times P)\,da$$

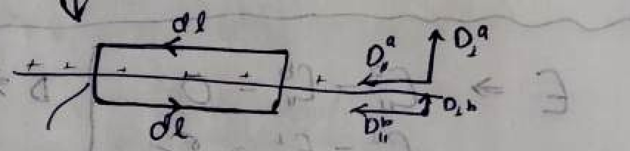
$$\int \vec{D}\cdot dl = \int \vec{P}\cdot dl$$

$$\int D_1''\cdot(-\hat{y})\cdot dl(-\hat{y}) + \int D_2''(-\hat{y})\cdot dl(\hat{y})$$

$$\boxed{D_a'' - D_b'' = P_a'' - P_b''}$$

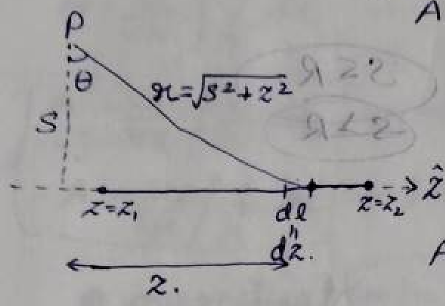
hence proved $\epsilon\varepsilon_0 + P = D$

from this we conclude that P and D have same direc



Vector potential due to a finite current carrying wire/line segment.

$$A = \frac{1}{4\pi}\int \frac{\mu_0 I}{r} dc. \Rightarrow \frac{1}{4\pi}\int \frac{\mu_0 I}{r} dl.$$



- $r = \sqrt{s^2+z^2}$
- $I = I\hat{z}$
- $dl = dc$

$$A = \frac{1}{4\pi}\int \frac{\mu_0 I \hat{z}}{\sqrt{s^2+z^2}} dz.$$

$$= \frac{\mu_0}{4\pi} I\hat{z} \int_{z_1}^{z_2} \frac{dz}{\sqrt{s^2+z^2}}$$

$z = s\tan\theta$

$dz = s\sec^2\theta\, d\theta$

$$\frac{\mu_0}{4\pi} I\hat{z} \int \frac{s\sec^2\theta\, d\theta}{\sqrt{s^2+s^2\tan^2\theta}} \Rightarrow \frac{\mu_0}{4\pi} I\hat{z}\int \frac{\sec^2\theta}{\sqrt{1+\tan^2\theta}} d\theta$$

$$\boxed{I\hat{z}\,\frac{\mu_0}{4\pi}\int \sec\theta\, d\theta}$$

$$\int \sec\theta\, d\theta = \ln\left[\tan\theta + \sec\theta\right]$$

$z = s\tan\theta$

$\tan\theta = \frac{z}{s}$

$\cos\theta = \frac{s}{\sqrt{s^2+z^2}}$

$\sec\theta = \frac{\sqrt{s^2+z^2}}{s}$

$$A = \frac{\mu_0}{4\pi} I\hat{z} \ln\left[\tan\theta + \sec\theta\right]$$

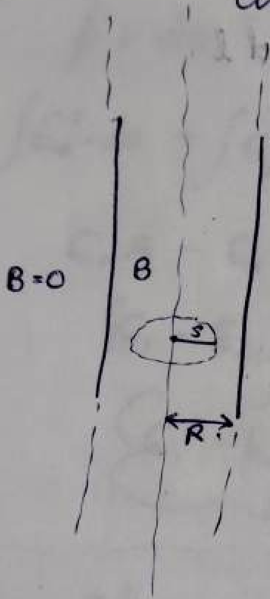
$$A = \frac{\mu_0}{4\pi} I\hat{z} \ln\left[\frac{z+\sqrt{s^2+z^2}}{s}\right]\Bigg|_{z_1}^{z_2}$$

$$= \frac{\mu_0}{4\pi} I\hat{z}\left\{\ln\left[\frac{z_2+\sqrt{s^2+z_2^2}}{s}\right] - \ln\left[\frac{z_1+\sqrt{s^2+z_1^2}}{s}\right]\right\}$$

$$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi}\hat{z}\left[\ln\frac{z_2+\sqrt{s^2+z_2^2}}{z_1+\sqrt{s^2+z_1^2}}\right)\hat{z}}$$

↳ direction of $\vec{A}$ is same as that of direct. of current in wire.

Q. find the vector poten. of an infinite solenoide with 'n' turns per unit length, radius 'r' and current I.



$B = \mu_0 I n$ (inside)
$= 0$ (outside).

$S \leq R$.
$S > R$

$\nabla \times A = B$.

$\int (\nabla \times A) da = \int B \cdot da$

$\oint A \cdot dl = \int B \cdot da$.

$\int B \cdot da = \mu_0 n I \int da$
$= \mu_0 n I \pi s^2$ — (1)

$\oint A \cdot dl = A \oint dl = A \cdot (2\pi s)$. — (11)

$\therefore$ on equa. (1) and (11).

$\mu_0 n I \pi s^2 = A(2\pi s)$

$\boxed{A = \frac{\mu_0 n I s}{2}}$ (inside).

Magnetization

Magnetization → surface bound current, volume bound current (due to)

not external when $B = 0$

$\vec{m} = 0$ → magnetic moment

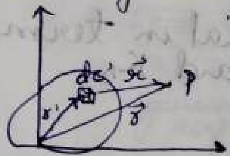
$$A = \frac{\mu_0}{4\pi}\int \frac{J}{\mathscr{r}}\,d\tau \quad \rightarrow \text{volume}$$

$$A = \frac{\mu_0}{4\pi}\int \frac{K}{\mathscr{r}}\,d\tau \quad \rightarrow \text{surface}$$

$$A = \frac{\mu_0}{4\pi}\int \frac{I}{\mathscr{r}}\,d\tau \quad \rightarrow \text{line}$$

only for finite value. not for infinite cases.

- magnetisat $(M) = \frac{m}{V}$ — volume



using multipole expansion

$$dA = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$M = \frac{m}{d\tau'}$$

$$\boxed{m = M\,d\tau'}$$

$$dA = \frac{\mu_0}{4\pi}\frac{M\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

$$A = \frac{\mu_0}{4\pi}\int M\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

$$\left\{\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = \nabla'\left(\frac{1}{\mathscr{r}}\right)\right\}$$

$$A = \frac{\mu_0}{4\pi}\int M\times\nabla'\left(\frac{1}{\mathscr{r}}\right)d\tau'$$

using identity $\{\nabla\times(fA) = f(\nabla\times A) - A\times(\nabla f)\}$

$$\boxed{\nabla'\times\left(\frac{1}{\mathscr{r}}M\right) = \frac{1}{\mathscr{r}}(\nabla'\times M) - M\left(\nabla'\left(\frac{1}{\mathscr{r}}\right)\right)}$$

$$A = \frac{\mu_0}{4\pi}\int\left\{\frac{1}{\mathscr{r}}(\nabla'\times M) - \nabla\times\left(\frac{1}{\mathscr{r}}M\right)\right\}d\tau'$$

$$= \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}(\nabla'\times M)\,d\tau' - \frac{\mu_0}{4\pi}\int\nabla'\times\left(\frac{1}{\mathscr{r}}M\right)d\tau'$$

using Advance theorem

$$\boxed{\int_V(\nabla\times V)\,d\tau = -\oint_S V\times da}$$

$$\int\nabla\times\left(\frac{1}{\mathscr{r}}M\right)d\tau' = -\oint\frac{1}{\mathscr{r}}M\times d\vec{a}'$$

$$= \frac{\mu_0}{4\pi}\int \frac{1}{r}(\nabla' \times M)\,d\tau' + \frac{\mu_0}{4\pi}\oint \left(\frac{M \times \hat{n}}{r}\right) da'$$

comparing with $A = \frac{\mu_0}{4\pi}\int \frac{J}{r}\,d\tau'$, $A = \frac{\mu_0}{4\pi}\int \frac{K}{r}\,d\tau'$

$\therefore$ $\boxed{J_b = \nabla' \times M}$ → volume bound current

$\boxed{K_b = M \times \hat{n}}$ → surface bound current

$\therefore$ $\boxed{A = \frac{\mu_0}{4\pi}\int \frac{J_b}{r}\,d\tau' + \frac{\mu_0}{4\pi}\oint \frac{K_b}{r}\,da'}$ — potential in terms of J_b and K_b.

• Diver $\boxed{\nabla \cdot B = 0}$

$\nabla \times B = \mu_0 J$

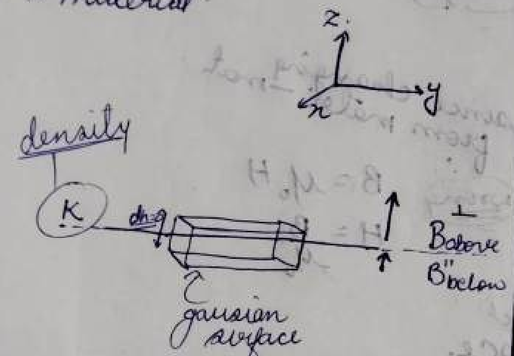
$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$$E_a^{\perp} - E_b^{\perp} = \sigma/\varepsilon_0$$
$$E_a^{\parallel} - E_b^{\parallel} = 0$$
$$D_a^{\perp} - D_b^{\perp} = \rho_f$$
$$D_a^{\parallel} - D_b^{\parallel} = P_a^{\parallel} - P_b^{\parallel}.$$

Boundary condition of magnetic field (B) and Vector potential (A).

$\boxed{B = \mu_0 H}$ → remains same from material to material

changes from material to material.



$\nabla \cdot B = 0$

$\int \nabla \cdot B \, dc = 0$

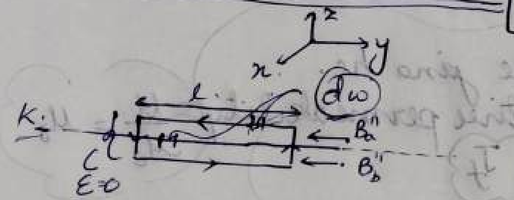
$\oint_S B \cdot da = 0$

$\int_1 B_a^{\perp} \cdot da + \int_2 B_b^{\perp} \cdot da = 0.$

$\int B_a^{\perp}(\hat{z}) da(\hat{z}) + \int B_b^{\perp}(\hat{z}) \cdot da(-\hat{z}) = 0.$

$B_a^{\perp} A - B_b^{\perp} A = 0$

$\boxed{B_a^{\perp} - B_b^{\perp} = 0}$ ∴ continous



$\boxed{I = K \, dw}$ line element

density

$\nabla \times B = \mu_0 J.$

$\int (\nabla \times B) da = \int \mu_0 J da.$

$\oint_C B \cdot dl = \mu_0 I_{inclosed}$

∴ magnetic field is discont.

$\int B_a'' dl + \int B_b'' dl = \mu_0 \int K dw.$

$\int B_a''(-\hat{y}) dl(-\hat{y}) + \int B_b''(\hat{y}) dl(\hat{y}) = \mu_0 K l.$

$\boxed{B_a'' - B_b'' = \mu_0 K.}$ → ∴ discontinous

or

$\boxed{B_a'' - B_b'' = \mu_0 (K \times \hat{n})}$ since K is surface ∴ it also has a normal component.

vector potential

$\boxed{A_a - A_b = 0}$ similar as in electros.

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- Magnetic susceptibility (χ) kai

dimension less
unit less.

$P \propto E$ (Polarisat $\propto$ extent of applied Ef).

$P = \chi_e E$ — electric suscep

$$\boxed{P = \varepsilon_0 \chi_e E}$$

similarly, $M \propto B$

$M = \frac{1}{\mu_0} \chi_m B$ → wrong

since changing from mater → mat.

$B = \mu_0 H$

$H = \frac{B}{\mu_0}$

$$\boxed{M = \chi_m H}$$ → correct

χ_m → magn. susce.

homework: yaha se $B = \mu_0 H$ take jana he.
find relation for relative permeability $\frac{\mu}{\mu_0} = \mu_r$
Relation b/w J_b and J_f