15/1/2024

DATE

Physics

Book Name

(EMT)

① Introduction to electrodynamics
- David J. Griffith
- Dr. K.K Tiwari
- Dr. Prasad
- S.P. Taneja / R.chand

Electrostatic

quark ⟶ nucleons ⟶ Nucleus ⟶ Atom ⟶ Molecules

quark → up, down

Charge

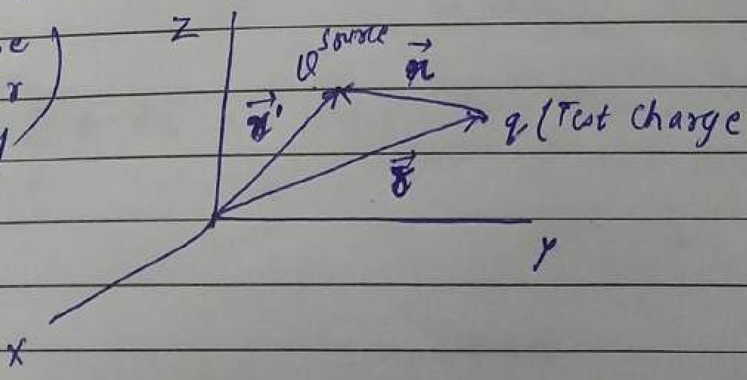
Charge is a physical property of any metal due to which it experienced electric and magnetic field.

Charge → +ve, −ve

- Source Charge (rest) (effect ~~ko~~ study kar rhe hai)
- Test charge (iske upper study kar rhe hai)

↓

(it may be moving or stationary)



$\vec{r'}$ = position vector of source charge
$\vec{r}$ = position vector of test charge
$\vec{r}$ = separation vector of source charge and test charge

According To Δ Law

$$\vec{r'} + \vec{r} = \vec{r}$$
$$\vec{r} = \vec{r} - \vec{r'}$$
$$|\vec{r}| = |\vec{r} - \vec{r'}|$$

Unit vector

$$\hat{r} = \frac{r}{|r|}$$

Conservation of charge

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He + Q$$

92C   90C + 2C

$$\gamma \rightarrow e^{\ominus} + e^{\oplus}$$

0   -1e   +1e

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Quantization of Charge

$\pm ne \quad e, 2e, 3e, \ldots$

$\nabla \rightarrow$ Del / operator

$\nabla \rightarrow \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

→ $\nabla \phi_{scalar} \rightarrow$ vector

↓

gradient / grad

∇ (vector)

dot   cross

→ $\nabla \cdot \underset{A}{(\text{vector})} \rightarrow$ scalar

↳ divergence / div.

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

→ $\nabla \times (\text{vector}) \Rightarrow$ Vector

↓

curl

(1) Divergence Theorem :- ∇

↳ surface Tension → vol. integral

Imp.

$$\oint_s \vec{E} \cdot da = \int_v (\nabla \cdot E)\, d\tau$$

(2) Stoke's Theorem $l \to s$ (Curl Theorem)

$$\oint_l \vec{E} \cdot d\vec{l} = \int_s (\nabla \times E)\, da$$

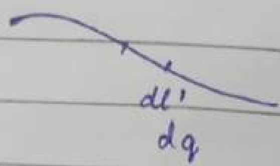
Note:-

Electric field is conservative (means independent of path/ consistance with respect to path)

Density

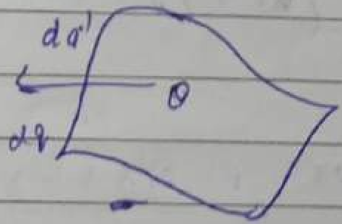
Charge density = $\frac{\text{charge}}{\text{line/area/volume length}}$

(1) Line (dl)
(2) surface (da)
(3) volume ($d\tau$)



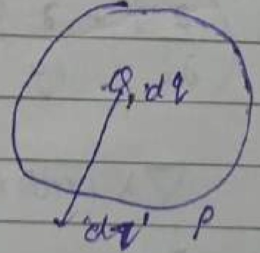
$$\lambda = \frac{dq}{dl'}$$

$$\boxed{dq = \lambda\, dl'}$$



$$\sigma = \frac{dq}{da'}$$

$$\boxed{dq = \sigma\, da'}$$



$$\rho = \frac{dq}{d\tau'}$$

$$\boxed{dq = \rho\, d\tau'}$$

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Electric field and Electrostatic potential ~~for~~ due to Charge distribution.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cdot\hat{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\cdot\hat{r}$$

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(\bar{r}')\cdot\hat{r}\,dl'}{r^2} \qquad (\because dq = \lambda\, dl')$$

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(\bar{r}')\cdot\hat{r}\cdot da'}{r^2} \qquad (\because dq = \sigma\, da')$$

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\bar{r}')\,\hat{r}\cdot d\tau'}{r^2} \qquad (\because dq = \rho\, d\tau')$$

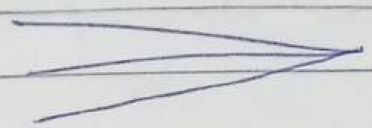
~~Electrical~~ Electric Potential

The amount of work needed to move a unit charge / test charge from infinity to a specific point against the electrostatic force.

Potential difference b/w ~~to~~ 2 points in electric field is work done per unit test charge in moving from one point to another point.

$\cancel{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$

$$\boxed{V_A - V_B = \frac{W_{BA}}{q_0}}$$



$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots \frac{q_n}{r_n}\right)$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

$$dq = \rho\, d\tau'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}')\,d\tau'}{r}$$

$$dq = \sigma\, da'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(\vec{r}')\,da'}{r}$$

$$dq = \lambda\, dl'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(\vec{r}')\,dl'}{r}$$

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Divergence and Curl of E

$\cancel{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$

$\cancel{\nabla \times E = 0}$

Divergence of E

Net electric flux $= \dfrac{q}{\epsilon_0}$

Imp. $\oint E \cdot da = \phi$

① Divergence theorem

$$\oint_s E \cdot da = \int_v (\nabla \cdot E)\, d\tau \quad \text{—— ①}$$

$$\oint_s E \cdot da = \frac{q}{\epsilon_0} \qquad \left(\because dq = \rho\, d\tau \quad q = \int \rho\, d\tau \right.$$

$$\oint_s E \cdot da = \int \frac{\rho\, d\tau}{\epsilon_0} \quad \text{—— ②}$$

$$\int_v (\nabla \cdot E)\, d\tau = \int \frac{\rho}{\epsilon_0}\, d\tau$$

$$\boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$$

Imp.

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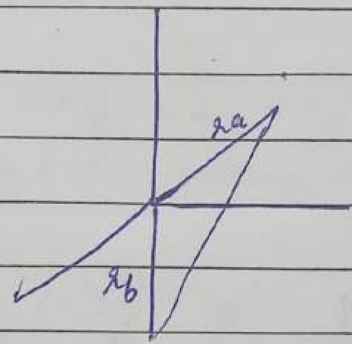
Curl of E $\boxed{\nabla \times E = 0}$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

$$\int \vec{E} \cdot dl = \frac{1}{4\pi\varepsilon_0} \int \frac{q}{r^2} dr$$

$$dl = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\, d\phi\, d\hat{\phi}$$

$$\int_a^b \vec{E} \cdot dl = \frac{1}{4\pi\varepsilon_0} \int \frac{q}{r^2} dr$$

$$= \left[\frac{q}{4\pi\varepsilon_0} \frac{1}{r} \right]_b^a$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r_a} - \frac{1}{r_b} \right]$$

$$\oint \vec{E} \cdot d\vec{l} = \int_a^b \vec{E} \cdot dl + \int_b^a E \cdot dl$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r_a} - \cancel{\frac{1}{r_b}} - \cancel{\frac{1}{r_a}} + \cancel{\frac{1}{r_b}} \right]$$

$$= 0$$

$$-\int E \cdot dl = \int (\nabla \times \vec{E})\, da$$

$$= 0$$

V. Imp.

Poisson's Eqⁿ & Laplace Eqⁿ

$$E = -\nabla V$$

$$\nabla \cdot E = -\frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot (-\nabla V) = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot (\nabla V) = -\frac{\rho}{\varepsilon_0}$$

$$\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}} \rightarrow \text{Poisson's equation}$$

Laplace eqⁿ

$$\boxed{\nabla^2 V = 0}$$

Conditions

$\rho = 0 \rightarrow$ Laplace equation

$\rho \neq 0 \rightarrow$ Poisson's equation

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Uniqueness Theorem

$\boxed{\nabla^2 V = 0} \rightarrow$ This is a unique solution

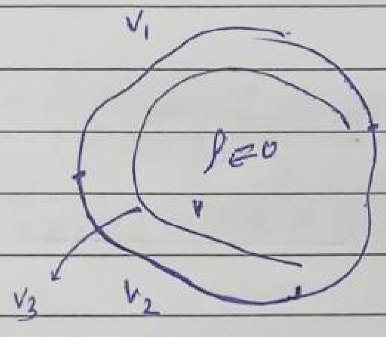
enclosed surface : $\rho_v = 0$

let there are two solution

V_1 & V_2

$\nabla^2 V_1 = 0$, $\nabla^2 V_2 = 0$

$V_3 = V_1 - V_2$



$V_1 - V_2 = 0$

$\boxed{V_1 = V_2}$

Outcomings :-

① In space it is difficult to identify the point of maxima and minima for an enclosed surface then obviously it is a unique solution.

② The solution is unique (i.e potential is same at boundaries it means no any maxima or minima) because there is no reference point inside ($\rho = 0$)

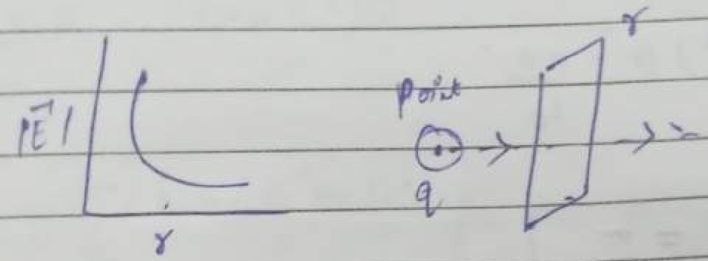
③ For propogation of EM waves uniqueness theorem or condition is important. $\boxed{\nabla^2 V = 0}$

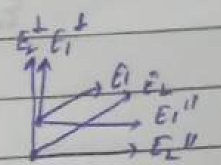
Red Wings

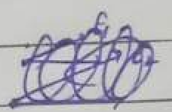
(4) Uniqueness ~~theorem~~ solution means there is only one solution.

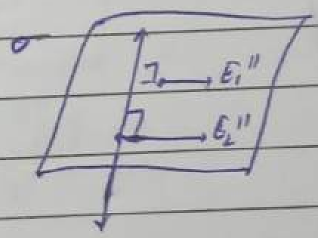
Boundary Condition for Electric field & Electric Potential.

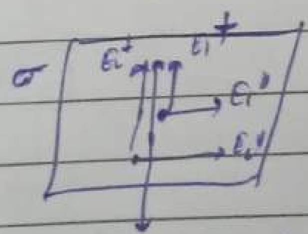
$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

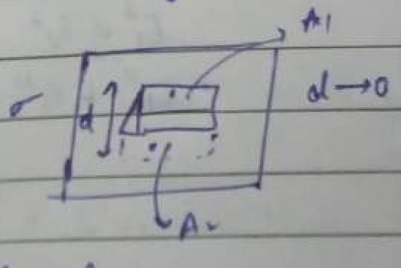












$$\oint \vec{E} \cdot da = \frac{q}{\varepsilon_0}$$

$$= \frac{\sigma A}{\varepsilon_0}$$

$A_1 = A_2 = A$

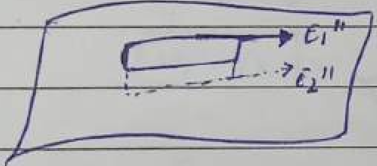
$$\int_{A_1} \vec{E_1} \cdot da + \int_{A_2} \vec{E_2} \cdot da = \frac{\sigma A}{\varepsilon_0}$$

$$E_1^{\perp} \cdot A + E_2^{\perp} \cdot A = \frac{\sigma A}{\varepsilon_0}$$

$$E_1^{\perp} \cdot A - E_2^{\perp} \cdot A = \frac{\sigma A}{\varepsilon_0}$$

$$(E_1^{\perp} - E_2^{\perp}) \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0}}$$

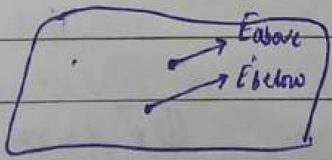


$$\oint \vec{E} \cdot dl = 0$$

$$\int E_1'' \cdot D + \int E_2'' \, dl = 0$$

$$E_2'' l - E_1'' l = 0$$

$$E_2'' l = E_1'' l$$



$$\left.\begin{matrix} E_1^{\perp} \ \& \ E_2^{\perp} \\ E_1'' \ \& \ E_2'' \end{matrix}\right\} \text{magnitude}$$

$\hat{n}$ & $\hat{p}$ along
$\perp$ $\parallel$

$\hat{n}$ is normal component and $\hat{p}$ is parallel component.

$E_{above} = E_1^{\perp}\hat{n} + E_1^{\parallel}\hat{p}$

$E_{below} = E_2^{\perp}\hat{n} + E_2^{\parallel}\hat{p}$

$E_{above} - E_{below}$

$E_1^{\perp}\hat{n} + \cancel{E_1^{\parallel}\hat{p}} - E_2^{\perp}\hat{n} + \cancel{E_2^{\parallel}\hat{p}}$ $\quad (\because E_1^{\parallel}\hat{p} = E_2^{\parallel}\hat{p})$

$E_1^{\perp}\hat{n} - E_2^{\perp}\hat{n}$

$\frac{\sigma}{\varepsilon_0}\hat{n} = (E_1^{\perp} - E_2^{\perp})\hat{n}$

$$\boxed{\frac{\sigma}{\varepsilon_0}\hat{n} = (E_1^{\perp} - E_2^{\perp})}$$

for potential

$$\boxed{d \to 0 \quad \begin{matrix} \cdot a \\ \cdot b \end{matrix}}$$

$V_b - V_a = \int_a^b E \cdot dl$

$V_b - V_a = 0$

$$\boxed{V_b = V_a}$$

$E = -\nabla V$

$\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}$

$-\nabla V_{above} + \nabla V_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}$

Multiply both side by $\hat{n}$

$$-\nabla V_{above}\,\hat{n} + \nabla V_{below}\,\hat{n} = \frac{\sigma}{\varepsilon_0}\,\hat{n}\,\hat{n}$$

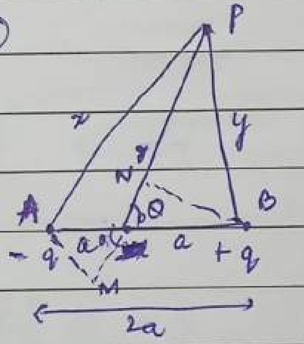
$$\nabla V_{above}\,\hat{n} - \nabla V_{below}\,\hat{n} = \frac{-\sigma}{\varepsilon_0}$$

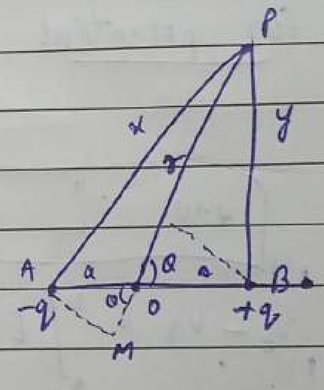
$$\frac{\partial V}{\partial n}_{above} - \frac{\partial V}{\partial n}_{below} = \frac{\sigma}{\varepsilon_0}$$

$$\boxed{\nabla V \cdot \hat{n} = \frac{\partial V}{\partial n}}$$

Electric Potential due to Electric dipole at any arbitary Point

$r >> a$





dipole moment $= q \times 2a$

dipole length $= 2a$

$$V = V_1 + V_2$$

$$V_1 = \frac{1}{4\pi\varepsilon_0}\,\frac{-q}{x}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{y}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{y} - \frac{1}{x}\right]$$

$$PM = PO + OM \qquad \frac{OM}{a} = \cos\theta$$

$$PM = r + a\cos\theta \qquad \boxed{OM = a\cos\theta}$$

$$PN = PO - ON$$

$$PN = r - a\cos\theta \qquad \frac{ON}{a} = \cos\theta$$

$$\boxed{ON = a\cos\theta}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + \cos\theta}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cancel{r} + a\cos\theta - \cancel{r} + a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

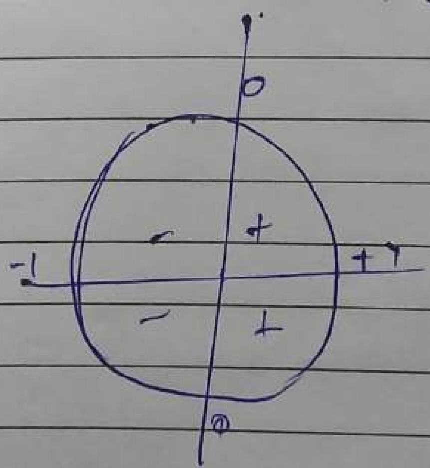
$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}}$$

$$p\cos\theta = p$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r}$$

$$\vec{p}\cdot\hat{r} = p \cdot 1\cos\theta = p\cos\theta$$



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Electric field due to electric dipole

$V = \frac{1}{4\pi\varepsilon_0} \cdot$

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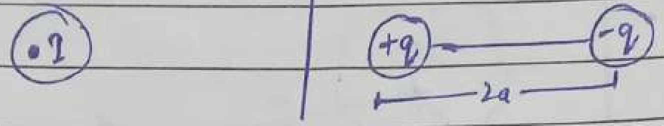
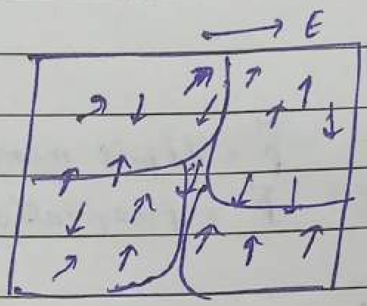
Bound Charges of electric polarization

non-polar | polar



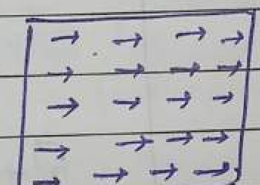
dipole length = 2a

dipole moment = $q \times 2a$



$P = 0$

$$\text{Polarization} = \frac{\text{dipole moment}}{\text{Volume}} = 0$$



$P \neq 0$

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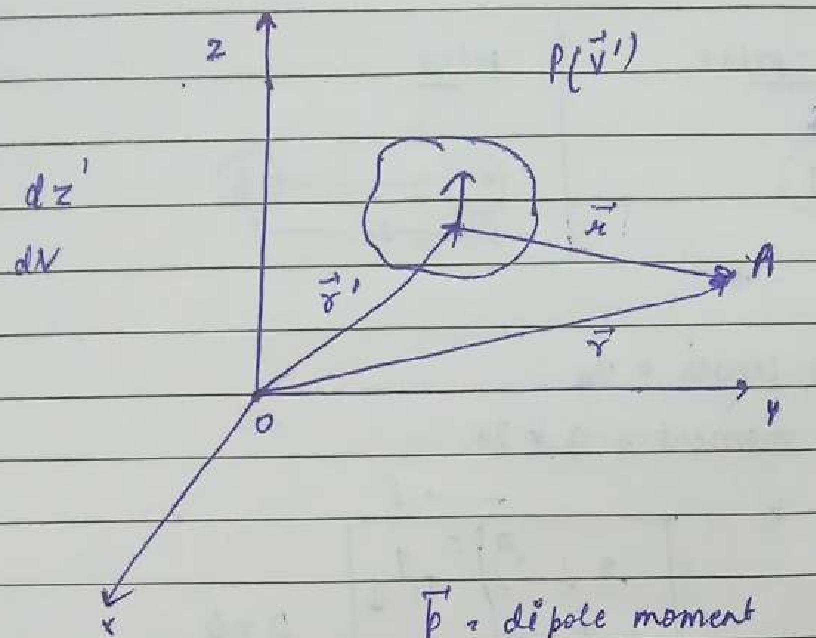
Bound charges due to electric polarization



$\vec{p}$ = dipole moment
$\vec{P}$ = Polarization

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\hat{\imath}}{\imath^2} \vec{p}$$

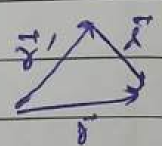
$$P = \frac{p}{d\tau'} \Rightarrow p = \vec{P} \cdot d\tau'$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\hat{\imath}}{\imath^2} P \cdot d\tau'$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\hat{\imath}}{\imath^2} P(\vec{r}')\, d\tau'$$

$$V = \frac{1}{4\pi\epsilon_0} \int P(\vec{r}') \frac{\hat{\imath}}{\imath^2}\, d\tau'$$

$$\nabla'\left(\frac{1}{\imath}\right) = \frac{\hat{\imath}}{\imath^2}$$



$$\vec{r}' + \vec{\imath} = \vec{r}$$

$$\vec{\imath} = \vec{r} - \vec{r}'$$

$$|\vec{\imath}| = |r - r'|$$

$$\nabla'\left(\frac{1}{\imath}\right) = \nabla'\left(\frac{1}{|r-r'|}\right)$$

$$= +1 \left(\frac{1}{|r-r'|}\right)^{+1}$$

$$= \left(\frac{1}{\imath^2}\right)\hat{\imath}$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \vec{P}(\vec{r}\,') \nabla'\left(\frac{1}{r}\right) d\tau'$$

Now apply product rule

$$\nabla \cdot (f\vec{A}) = f\, \nabla \cdot \vec{A} + A \cdot \nabla f$$

$$\nabla \cdot \left(\frac{1}{r} \vec{P}(\vec{r}\,')\right) = \frac{1}{r} \nabla \cdot \vec{P}(\vec{r}\,') + \vec{P}(\vec{r}\,') \cdot \nabla\left(\frac{1}{r}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \nabla \cdot \left(\frac{1}{r} \vec{P}(\vec{r}\,')\right) d\tau' - \frac{1}{4\pi\varepsilon_0} \int \frac{1}{r} \nabla \cdot P(\vec{r}\,') d\tau'$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{1}{r} \vec{P}(\vec{r}\,') da - \frac{1}{4\pi\varepsilon_0} \int \frac{1}{r} \nabla \cdot P(\vec{r}\,') d\tau'$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\vec{r})}{r} da$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r})}{r} d\tau$$

$$\boxed{\begin{aligned} \sigma_b &= \vec{P} \cdot \vec{n} \\ \rho_b &= -\nabla \cdot \vec{P} \end{aligned}}$$

$$\int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \hat{n}\, da$$

Red Wings

Electric displacement

Boundary conditions for electric Displacement vector