Roll No. 2200101300l

Total Pages : 4

**013202**

**July, 2023**

**B.Tech. (ME) - IInd SEMESTER**

**Calculus, ODE and Complex Variables**

**(BSC-106A/BSCH-106A)**

Time: 3 Hours] [Max. Marks. : 75

*Instructions :*

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

## PART–A

**1.** (a) State Liouville's Theorem. (1.5)

(b) State Cauchy Residue theorem. (1.5)

(c) Show that the function sin $z$ is analytic in the finite $z$-plane. (1.5)

(d) Show that the transformation $w = e^z$ is always conformal. (1.5)

(e) Find the solution of the differential equation

$$\frac{dy}{dx} - y = 5 \sin x. \qquad (1.5)$$

(f) What is the general form of the Bessel's differential equation? (1.5)

(g) Find the integrating factor of the following differential equation : (1.5)

$$(x^3 + xy^4)\,dx + 2y^3\,dy = 0.$$

(h) State Green's Thorem. (1.5)

(i) Evaluate the integral

$$\int_0^1 \int_1^2 x(x+y)\,dy\,dx. \qquad (1.5)$$

(j) Find the general solution of the following differential equation :

$$y = (x-a)p - p^2, \text{ where } p = \frac{dy}{dx}. \qquad (1.5)$$

## PART–B

2. (a) Evaluate the integral $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ by changing the order of integration. (7)

(b) Evaluate the integral $\iint_S \vec{F}.\vec{n}\,ds$ if $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is part of the surface $x^2 + y^2 + z^2 = 1$, which lies in the first octant. (8)

3. (a) Find the General solution of the differential equation $y'' + y = \text{cosec } x \cot x$ using the method of variation of parameters. (7)

(b) Find the series solution in series of power of $x$ of the following differential equation :

$$4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0. \quad (8)$$

4. (a) Solve the following differential equation :

$$p^2 + 2xp - 3x^2 = 0$$

where $p = \frac{dy}{dx}$. (7)

(b) Solve the differential equation :

$$(2x - y)dy + (2y + x)\,dx. \quad (8)$$

5. (a) Show that the function $u(x, y) = 2x + y^3 - 3x^2y$ is harmonic. Find its conjugate harmonic function $v(x, y)$ and the corresponding analytic function $f(z)$. (7)

(b) Find the image of the closed half disk

$$|z| \leq 1, \; Im\; z \geq 0,$$

under the bilinear transformation (8)

$$w = \frac{z}{z+1}.$$

6. (a) Evaluate the integral $I = \int_0^{2\pi} \frac{d\theta}{2+\sin\theta}$. (8)

(b) Find the residue at all the singular points of the following functions : (7)

$$f(z) = \frac{1}{z^3 + z^5},$$

7. (a) Expand the function $f(z) = 1/z$ about $z = 2$ in Taylor's series. Obtain its radius of convergence. (7)

(b) State and prove Stoke's Theorem. (8)

---