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Roll No. ....

Total Pages : 05

013202

May 2025

B.Tech. (Mechanical Engineering)

(Second Semester)

MATHEMATICS - II (Calculus, Ordinary Differential Equations and Complex Variable)

(BSC-106A/MTU-146-V)

Time : 3 *Hours*] [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) Evaluate $\iint xy(x+y)dxdy$ over the region bounded by the line $y = x$ and the curve $y = x^2$. **1.5**

   (b) State "Gauss Theorem". **1.5**

(c) Determine order and degree of the following differentiable equation : 1.5

$$\left(\frac{dy}{dx}\right)^3 + \frac{y}{x} = \sin x$$

(d) Solve : 1.5

$$p = \sin(y - xp).$$

(e) Evaluate : 1.5

$$\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0$$

(f) Find solution of the given differential equation : 1.5

$$\frac{d^2y}{dx^2} = x + \log x.$$

(g) Find real and imaginary part of the function $f(z) = \sin z$. 1.5

(h) Let $f(z)$ be a function with domain D. If $f(z)$ and its conjugate $\overline{f(z)}$ both are analytic in D, then show that $f(z)$ is a constant function. 1.5

(i) State "Maximum-Modulus Theorem". 1.5

(j) If $f(z)$ is an analytic function at $z = a$ and it has a zero of order m at $z = a$, then $f(z)$ can be expressed in the form $f(z) = (z - a)^m \phi(z)$, where $\phi(z)$ is analytic at a and $\phi(a) = 0$. 1.5

Part B

2. (a) Change the order of integration in the following integral and evaluate : 7

$$\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$$

(b) Verify Green's theorem for $\int_C \left[(xy + y^2)dx + x^2dy\right]$, where C is the boundary, of the area between $y = x^2$ and $y = x$. 8

3. (a) (i) Solve : 8

$$p^2 + 2py \cot x - y^2.$$

(ii) Find the general solution of $y = 2px + y^2p^3$.

(b) Solve : 7

$$(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0.$$

4. (a) Solve by the method of variation of parameters : 8

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x.$$

(b) Find the series solution about $x = 0$ of the differential equation : 7

$$(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - y = 0$$

5. (a) Show that analytic function with constant argument is constant. 7

(b) Determine the analytic function $f(z) = u + iv$, if : 8

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)} \text{ and } f\left(\frac{\pi}{2}\right) = 0$$

6. (a) Find the image of the circle $|z - 2i| = 2$ under the mapping : 7

$$w = \frac{1}{z}$$

(b) If $F(\alpha) = \oint_C \frac{4z^2 + z + 5}{z + \alpha} dz$, where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, find the value of : 8

(i) $F(3.5)$

(ii) $F(i)$

(iii) $F'(-1)$

(iv) $F''(-i)$

7. (a) Evaluate : 5

$$\lim_{z \to 0}\left[\frac{1}{1-e^z}\int_C \frac{d\alpha}{1+\alpha^2}\right],$$

where C is the straight line path from $\alpha = 0$ to $\alpha = z$ and z is a point in the neighbourhood of the origin.

(b) Using the method of contour integration, prove that : 10

$$\int_0^{2\pi} \frac{1}{1 - 2a\cos\theta + a^2} d\theta = \begin{cases} \frac{2\pi}{1-a^2}, & \text{if } |a| < 1 \\ \frac{2\pi}{1-a^2}, & \text{if } |a| > 1 \end{cases}$$

where a is a complex constant.

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