Time: 3 Hours

Max. Marks:75

Instructions:
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
2. Answer any four questions from Part -B in detail.
3. Different sub-parts of a question are to be attempted adjacent to each other.

PART-A

Que.1(a)Evaluate $\int_0^1 \int_y^{y^{?}+1} x^2 y dx dy$

(b) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. (1.5)

(1.5)

(c) Solve $(xy^3+y)dx+(2x^2y^2+x+y^4)dy = 0$.

(d) Solve the differential equation:
$y = 2px+p^4x^2$ (solvable for y). (1.5)

(e) Solve $(D^4+6D^2+9) y = 0$, where D= d/dx. (1.5)

(f) Write the Bessel's differential equation of order n. (1.5)

(g) State C-R Equations. (1.5)

(h) Define conformal mapping. (1.5)

(i) State Cauchy's integral theorem and Cauchy's integral formula. (1.5)

(j) State Cauchy's Residue Theorem. (1.5)

PART-B

Que.2 (a)Change the order of integration in the given integral and then evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$ (7)

(b)Verify the Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = 0, y = 0, x + y = 1$. (8)

Que.3 (a)Solve the differential equation $(2y\sin x+3y^4\sin x\cos x)dx - (4y^3\cos^2x+\cos x)dy = 0$. (7)

(b)Solve the differential equation: $y = 2px+y^2p^3$ (Solvable for x). (8)

Que.4 (a)Using variation of parameter ,solve $(D^2- 6D + 9)y = \frac{e^{3x}}{x^2}$, where D = d/dx (7)

(b)Express $4x^3- 2x^2- 3x + 8$ in terms of Legendre's polynomial. (8)

Que.5 (a)Show that the function $u = e^{-2xy}\sin(x^2-y^2)$ is harmonic. Find the conjugate function 'v' and express u+iv as an analytic function of z. (7)

(b)Under the transformation $w = 1/z$, find the image of the given curve: $| z - 2i | = 2$ (8)

Que.6 (a) Expand $\frac{e^{2z}}{(z-1)^3}$ about the singularity $z = 1$ in Laurent's series. (7)

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$ using Residue theorem. (8)

Que.7 (a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$ (7)

(b) Find the sum of the residues of the function $f(z) = \frac{\sin z}{z\cos z}$ at its poles inside the circle $|z| = 2$ (8)