Roll No. 2200100800

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B.Tech. 2nd Semester

MATHEMATICS-II

(Calculus, Ordinary Differential Equations and Complex Variable)

(BSC-106D)

Time: 3 Hours] [Max. Marks. : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) State Gauss Divergence Theorem. (1.5)

   (b) Evaluate $\int_0^1 \int_x^{\sqrt{x}} \left(x^2 + y^2\right) dx\, dy$: (1.5)

   (c) Solve $(x + 1) \dfrac{dy}{dx} - y = e^{3x}(x+1)^2$. (1.5)

   (d) Solve the differential equation $p = \log(px - y)$. (1.5)

(e) Express the given differential equation

$$(x^2D^2 + xD + 7)\, y = 2/x$$

into linear differential equation with constant coefficient and find its complementary function. (1.5)

(f) Define Bessel's function of first kind. (1.5)

(g) Define Mobius transformations. (1.5)

(h) Is function $f(z) = \bar{z}$ is analytic or not? Explain. (1.5)

(i) Give the statement of Liouville's Theorem. (1.5)

(j) What type of singularity have the function. (1.5)

$f(z) = \frac{e^{1/z}}{z^2}$

PART–B

2. (a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ by changing to polar coordinates. Hence show that

$$\int_0^\infty e^{-x^3} dx = \sqrt{\pi}/2.$$ (8)

(b) Verify Stoke's theorem for $F' = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by the lines $x = \pm a,\ y = 0,\ y = b$. (7)

3. (a) Solve $(xy^2 - e^{1/x3})\, dx - x^2\, y\, dy = 0$. (7)

(b) Solve the differential equation $y - 2px = \tan - 1(xp^2)$. (Solvable for y). (8)

$\tan^{-1}(xp^2)$

4. (a) Solve in series the equation

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0.$$

(8)

(b) Solve by using the method of variation of parameters,

$$y''-2y'+y=e^x \log x.$$

(7)

5. (a) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Hence find the image of $|z| < 1$.

(8)

(b) Determine the analytic function whose real part is $u = e^{2x}(x \cos 2y - y \sin 2y)$.

(7)

6. (a) Determine the poles of the function

$$f(z)=\frac{z^2}{(z-1)^2(z+2)}$$

and the residue at each pole. Hence evaluate

$$\oint_C f(z)\,dz$$

where C is the circle $|z| = 2.5$.

(8)

(b) Find the Laurent's expansion of

$$f(z)=\frac{7z-2}{(z+1)z(z-2)}.$$

in the region $1 < z + 1 < 3$.

(7)

7. (a) By integrating around a unit circle, evaluate

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta .$$ (8)

(b) Evaluate $\int_0^1 \int_{e^x}^{e} \frac{dy\, dx}{\log y}$ by changing the order of integration. (7)
