July 2023
B. Tech. - II SEMESTER
Mathematics II (Probability and Statistics)
(BSC-106E/BSCH-106E)

Time : 3 Hours]   [Max. Marks : 75

Instructions :

   1.   *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*

   2.   *Answer any four questions from Part-B in detail.*

   3.   *Different sub-parts of a question are to be attempted adjacent to each other.*

   4.   *The candidate is required to attempt the question paper in the language as per his/her medium of instruction.*

   5.   *No table is needed. All the tabulated values required are mentioned in the question paper.*

PART-A

1.   (a)   What is the probability of throwing a number less than 6 with an unbiased dice?   (1.5)

     (b)   Define random experiment with example.   (1.5)

(c) If five coins are tossed, what is the probability that all will show a head? (1.5)

(d) Fill in the blank : Correlation coefficient is the _____ between the regression coefficients. (1.5)

(e) Find $E(Y)$, given $E(X) = 3$ and $Y = \dfrac{x}{3} - 3$, $E(X)$ denotes expectation of the random variable X. (1.5)

(f) If X follows Poisson distribution with parameter λ, then write its moment generating function. (1.5)

(g) Find the standard deviation of the Binomial distribution $B\left(16, \dfrac{1}{4}\right)$. (1.5)

(h) Let $X \sim N(2, 9)$. Find out μ_{13} where μ_{13} is the 13th order moment about mean. (1.5)

(i) Define Two-tailed test. (1.5)

(j) Write the critical value for a right-tailed test at 1% level of significance. (1.5)

PART–B

2. (a) What is the probability that a leap year, selected at random, will have 53 Sundays? (7)

(b) Out of the numbers 1 to 120, one is selected at random. What is the probability that the number is divisible by 8 or 10? (8)

3. (a) There are 4 boys and 2 girls in room A and 5 boys and 3 girls in room B. A girl from one of the room laughed loudly. What is the probability that the girl who laughed is from room B? (7)

(b) Two discrete random variables X and Y have the joint probability density function

$$p_{X,Y}(x,y) = \frac{\lambda^x e^{-\lambda} k^y (1-k)^{x-y}}{y!(x-y)!},$$

$$y = 0, 1, 2, \dots x; \quad x = 0, 1, 2, \dots$$

where λ and k are constants with $\lambda > 0$ and $0 < k < 1$. Find the marginal probability density functions of X and Y. (8)

4. (a) A random variable X takes the values $-1, 1, 3, 5$ with associated probabilities $1/6$, $1/6$, $1/6$ and $1/2$. Find $P(|X-3| \geq 1)$ by direct computation. Also find an upper bound of this probability by using Chebychev's inequality. (7)

(b) It is given that 3% of the electric bulbs manufactured by a company are defective. Using Poisson distribution, find the probability that a sample of 100 bulbs will contain no defective bulb. (Given $e^{-3} = 0.05$) (8)

5. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and the standard deviation of the normal distribution. (Given that $P(0 \le z \le 0.496) = 0.19$, $P(0 \le z \le 1.405) = 0.42$).

(7)

(b) For 10 observations on price (x) and supply (y), the following data are obtained (in approximate units) :

$\Sigma x = 130$, $\Sigma y = 220$, $\Sigma x^2 = 2288$, $\Sigma y^2 = 5506$, $\Sigma xy = 3467$. Find the line of regression of y on x and estimate the supply when the price is 16 units. (8)

6. (a) If a random sample of size 20 from a normal population with standard deviation 5.2 shows a mean of 16.9, test at 5% level of significance that the sample is drawn from a population with mean 15.5. Also calculate 99% confidence limit for mean. (7)

(b) A company has two branches at Delhi and Mumbai. The director wanted to know if the workers at the two places would like a new plan of work and a survey was conducted for this reason. Out of a sample of 500 workers in Delhi, 62% favoured the new plan. At Mumbai, out of a sample of 400 workers, 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level? (8)

7. (a) 4 coins are tossed together 160 times and following results are obtained :

No. of heads	0	1	2	3	4
Observed frequencies	17	52	54	31	6

Under the assumption that coins are balanced, find the expected frequencies of getting 0, 1, 2, 3, 4 and test goodness of fit at 5% level. (Given $\chi^2_{0.05} = 9.488$) Ⓤ

(7)

(b) A fertilizer mixing machine is set to give 12 kg of nitrate for every quintal bag of fertilizer. Ten bags, each of weight 100 kg, are examined and the percentage of nitrate are as follows :

11, 14, 13, 12, 13, 12, 13, 14, 11, 12 Ⓧ

Is there reason to believe that the machine is defective? (Given that tabulated value of $t_{0.05}$ for 9 d.f. is 2.262).

(8)

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