PFQ - PYQFort Question-Bank

Questions till 2023 (Clickable)

Mathematics (BSC-106E)

Probability & Statistics

MODULE 1: BASIC PROBABILITY

Probability Spaces (Basic Probability):

- Define random experiment with example. (1.5 marks)
- What is the probability of throwing a number less than 6 with an unbiased dice? (1.5 marks)
- If two dice are thrown together, what is the probability that the sum is neither 7 nor 11? (1.5 marks)
- Find the probability of drawing any one spade card from a full pack of cards. (1.5 marks)
- If five coins are tossed, what is the probability that all will show a head? (1.5 marks)
- Out of the numbers 1 to 120, one is selected at random. What is the probability that the number is divisible by 8 or 10? (8 marks)
- What is the probability that a leap year, selected at random, will have 53 Sundays? (7 marks)
- Suppose that an urn contains 8 red balls and 4 white balls. We draw 2 balls from the urn without replacement. If we assume that at each draw each ball in the urn is equally likely to be chosen, what is the probability that both balls drawn are red? (2 marks)
- If Var(X) = 2, then calculate Var(3X - 2). (1.5 marks)
- Suppose that a particular trait of a person is classified on the basis of one pair of genes and suppose that 'd' represents a dominant gene and 'r' represents a recessive gene. Thus, a person with 'dd' genes is pure dominance, one with 'rr' is pure recessive and one with rd is hybrid. The pure dominance and the hybrid are alike in appearance. Children receive one gene from each parent. If with respect to a particular trait, 2 hybrid parents have a total of 4 children, what is the probability that 3 of the 4 children have the outward appearance of the dominant gene? (3 marks)

Independence of Events:

- If E and F are independent events, prove that E and F^c are independent events. (2 marks)
- If X and Y are two mutually independent events, then find Cov(X, Y). (1.5 marks)

Conditional Probability:

- A and B are two weak students of statistics and their chances of solving a problem correctly are 1/6 and 1/8 respectively. If the probability of their making a common error is 1/525 and they obtain the same answer, find the probability that their answer is correct. (8 marks)
- A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. What is the chance that actually there was six? (1.5 marks)
- In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% students fail in both. One student is chosen at random. What is the probability that (a) he fails in Mathematics if he has already failed in Physics (b) he fails in Physics or Mathematics? (7 marks)
- There are 4 boys and 2 girls in room A and 5 boys and 3 girls in room B. A girl from one of the rooms laughed loudly. What is the probability that the girl who laughed is from B? (7 marks)

Discrete Random Variables:

- A die is tossed thrice. Getting '5' or '6' on the die in a toss is taken as success. Find the mean of number of successes. (1.5 marks)
- A random variable X has the following probability function:

Values of X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k, (ii) Evaluate P (X < 6), P ($X \geq 6$), and P (0 < X < 5) (iii) If P ($X \leq a$) $> \frac{1}{2}$, find the minimum value of a, and (iv) Determine the distribution function of X. (8 marks)

Poisson Distribution:

- If X follows Poisson distribution with parameter λ, then write its moment generating function. (1.5 marks)

Binomial Distribution:

- Find the standard deviation of the Binomial distribution B(16, 1/4). (1.5 marks)

- Ten percent of the tools in a manufacturing process turn out to be defective. Find the probability that in sample of 10 tools chosen at random, exactly two will be defective using binomial distribution. (7 marks)

Expectation and Moments:

- Find E(Y), given E(X)=3 and Y = $\frac{x}{3} - 3$, E(X) denotes expectation of the random variable X. (1.5 marks)
- Let X $\sim$ N(2, 9). Find out μ_{13} where μ_{13} is the 13th order moment about mean. (1.5 marks)

Chebyshev's Inequality:

- Define Chebyshev's Inequality. (1.5 marks)
- A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes using Chebyshev's inequality. (7 marks)
- A random variable X takes values -1, 1, 3, 5 with probabilities 1/6, 1/6, 1/6, and 1/2. Find P($|X - 3| \geq 1$) by direct computation. Also find an upper bound of this probability by using Chebychev's inequality. (7 marks)

MODULE 2: CONTINUOUS PROBABILITY DISTRIBUTION

Continuous Random Variables:

- Define exponential random variable. If X is exponential with parameter λ > 0, compute E[X], Var(X) and cumulative distribution function F(a), $a \geq 0$ (4 marks)
- Let X be a continuous random variable with p.d.f.:

$$f(x) = \begin{cases} ax, & 0 \leq x \leq 1 \\ a, & 1 \leq x \leq 2 \\ -ax + 3a, & 2 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

  (i) Determine the constant a (ii) Compute P (X $\leq$ 1.5). (8 marks)
- A continuous random variable X follows the probability law $f(x) = A(x + 1)$, $0 \leq x \leq 1$. Determine A. (1.5 marks)

Normal Distribution:

- Compute the moment generating function of unit normal random variable. (4 marks)

- X be a normal variate with mean 30 and S.D. 5. Find probabilities that (i) $26 \leq X \leq 40$, (ii) $X \geq 45$ (iii) $|X - 30| \geq 5$ (7 marks)
- Write formula for mode and median of Normal Distribution. (1.5 marks)
- Derive the expression of Mode for Normal distribution. (3 marks)
- In a normal distribution, 31% items are under 45 and 8% are over 64. Find mean and standard deviation of the distribution ? Given that P($0 \leq z \leq 0.496$) = 0.19, P($0 \leq z \leq 1.405$) = 0.42 (4 marks)
- The mean-yield for a one-acre plot is 662 kilos with s.d. 32 kilos. Assuming normal distributions, how many one-acre plots in a batch of 1000 plots would you expect to have yield (a) over 700 kilos and (b) below 650 kilos? (Given that the P($0 < Z < z$) = 0.3830 when z=1.19 and P($0 < Z < z$) = 0.1430 when z=0.38) (8 marks)

Gamma Distribution:

- Find variance of Gamma Distribution. (1.5 marks)
- Determine the moment generating function of Gamma distribution. (7 marks)

MODULE 3: BIVARIATE DISTRIBUTIONS

Bivariate Distributions and Properties:

- Prove that (X, Y) possesses bivariate normal distribution if every linear combination of X, Y is normal variate. (1.5 marks)
- If X and Y are independent binomial random variables with identical parameters n and p, calculate conditional expected value of X given X + Y = m. (3 marks)
- Two discrete random variables X and Y have the joint probability density function

$$p_{X,Y}(x,y) = \frac{\lambda^x e^{-\lambda} k^y (1-k)^{x-y}}{y!(x-y)!}, \quad y = 0, 1, 2, \ldots, x;\ x = 0, 1, 2, \ldots$$

  Where λ and k are constants with $\lambda > 0$ and $0 < k < 1$. Find the marginal probability density functions of X and Y. (8 marks)
- Define marginal probability for discrete random variable. (1.5 marks)
- Let X and Y be jointly distributed with the probability density function:

$$f_{X,Y}(x,y) = \begin{cases} 2 - x - y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

  Are X and Y independent? (8 marks)

MODULE 4: BASIC STATISTICS

Measures of Central Tendency:

- Find the recurrence relation for the moments about mean of binomial distribution with parameters n and p. Hence, find the coefficient of skewness β_1 (4 marks)
- For a distribution with mean 10, variance 16, $\gamma_1 = +1$, $\beta_2 = 4$, obtain first four moments about origin, i.e., zero. Comment upon the nature of distribution. (8 marks)
- Obtain recurrence relation for moments about mean of binomial distribution. Find skewness and kurtosis coefficients. (4 marks)

Binomial Distribution Properties:

- Find the mode of discrete distribution B(16, 1/4). (1.5 marks)
- If X is a binomial random variable with parameters (n, p); where $0 < p < 1$, then show that as k goes from 0 to n. P $\{X = k\}$ first increases monotonically and then decreases monotonically, reaching its largest value when k is the largest integer less than or equal to (n + 1) p. (4 marks)
- Fit a Binomial distribution to the following frequency distribution:

X	0	1	2	3	4	5	6
F	13	26	52	58	32	16	4

  (7 marks)
- In binomial distribution with 5 independent trials, P(1 success) = 0.4096, P(2 successes) = 0.2048. Find parameter p for the distribution. (7 marks)

Poisson Distribution Properties:

- Define Poisson random variable. Let X be a Poisson random variable with parameter λ, $\lambda > 0$. Find E[X], E[X^2], Var(X) and SD(X) (4 marks)
- Show that for Poisson distribution with unit mean, Mean deviation about Mean is (2/e) times standard deviation. (3 marks)
- If X has a Poisson distribution such that P (X = 2) = $\frac{2}{3}$ P (X = 1), evaluate (a) P (X = 0) and (b) P (X = 3). (7 marks)

Correlation and Regression:

- What is scatter diagram? (1.5 marks)
- Define covariance of two random variables. (1.5 marks)
- Correlation coefficient is the ____ between regression coefficients. (1.5 marks)

- Show that if X_1 and X_2 are standard normal variates with correlation coefficient ρ between them, then the correlation coefficient between X_1^2 and X_2^2 is given by ρ^2. (8 marks)
- Let (X, Y) have the joint p.d.f. is given by:

$$f(x,y) = \begin{cases} 1, & \text{if } |y| < x,\ 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

  Show that the regression of Y on X is linear but regression of X on Y is not linear. (7 marks)
- Regression equations: 3x + 2y = 26 and 6x + y = 31. Find mean values and correlation coefficient between x and y. (3 marks)
- For 10 observations on price (x) and supply (y), the following data are obtained (in approximate units): $\Sigma x = 130, \Sigma y = 220, \Sigma x^2 = 2288, \Sigma y^2 = 5506, \Sigma xy = 3467$. Find the line of regression of y on x and estimate the supply when the price is 16 units. (8 marks)

MODULE 5: APPLIED STATISTICS

Curve Fitting - Straight Lines:

- Fit a straight line to the following data:

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

  (4 marks)
- Fit a straight line to the following data:

x	0	1	2	3	4
y	1.0	1.8	3.3	4.5	6.3

  Hence find the difference between the actual value and obtained value of y when x = 3. (8 marks)

Curve Fitting - Second Degree Parabolas:

- Find least squares approximation of second degree for discrete data:

x	-2	-1	0	1	2
y	15	1	1	3	10

  (8 marks)

Test of Significance:

- Define Two-tailed test. (1.5 marks)

- Define Null hypothesis and Confidence Limits. (1.5 marks)
- A random sample of 200 measurements from a large population gave a mean value of 50 and standard deviation of 9. Determine 95% confidence interval for the mean of population (3 marks)
- Write critical value for right-tailed test at 1% significance level. (1.5 marks)
- For a right-tailed large sample test, which is the critical value at 5% level of significance? (1.5 marks)
- Define Type I error with example. (1.5 marks)

Large Sample Tests:

- **Single Mean:** If a random sample of size 20 from a normal population with standard deviation 5.2 shows a mean of 16.9, test at 5% level of significance that the sample is drawn from a population with mean 15.5. Also calculate 99% confidence limit for mean (7 marks)
- **Single Proportion:** In a sample of 1000 people in Haryana, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state? (Test at 1% level) (8 marks)
- **Difference of Means:** The average hourly wage of a sample of 150 in a plant 'A' was Rs 2.56 with a standard deviation of Rs. 1.08. The average wage of a sample of 200 workers in plant 'B' was Rs 2.87 with a standard deviation of Rs 1.28. Can an applicant safely assume that the hourly wages paid by plant 'B' are higher than those paid by plant 'A'? (3 marks)
- **Difference of Proportions:** A company has two branches at Delhi and Mumbai. The director wanted to know if the workers at the two places would like a new plan of work and a survey was conducted for this reason. Out of a sample of 500 workers in Delhi, 62% favoured the new plan. At Mumbai, out of a sample of 400 workers, 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level? (8 marks)
- A random sample of size 16 values from a normal population showed a mean of 53 and a sum of squares of deviation from the mean equals to 150. Can this sample be regarded as taken from the population having 56 as mean? Obtain 95% and 99% confidence limits of the mean of the population. (7 marks)
- The heights of six randomly chosen sailors are (in inches): 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers. (8 marks)
- The intelligence test of two groups of boys and girls gives the following results: (4)

Girls	Mean = 84	S.D. = 10	N = 121
Boys	Mean = 81	S.D. = 12	N = 81

(a) Is the difference in mean scores significant?
(b) Is the difference between the standard deviations significant? (4 marks)

MODULE 6: SMALL SAMPLES

Test for Single Mean:

- The mean weekly sales of soap bars in departmental stores were 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful? [Given that tabulated value of $t_{0.05}$ for 21 d.f. is 1.72] (8 marks)
- A fertilizer mixing machine is set to give 12 kg of nitrate for every quintal bag of fertilizer. Ten bags, each of weight 100 kg, are examined and the percentage of nitrate are as follows:

  11, 14, 13, 12, 13, 12, 13, 14, 11, 12

  Is there reason to believe that the machine is defective? (Given that tabulated value of $t_{0.05}$ for 9 d.f. is 2.262). (8 marks)

Test for Difference of Means:

- The means of two single large samples of 1000 and 2000 members are 67.5 inches and 63.0 inches, respectively. Can the samples be regarded as drawn from the same population of s.d. 2.5 inches? (Test at 5% level) (7 marks)

Test for Ratio of Variances:

- The yield of wheat in a random sample of 1000 farms in a certain area has a standard deviation of 192 kg. Another sample of 1000 farms gives a standard deviation of 224 kg. Are the standard deviations significantly different? (8 marks)

Chi-square Tests:

- A set of five similar coins is tossed 320 times and the result is

No. of heads:	0	1	2	3	4	5
Frequency:	6	27	72	112	71	32

  Test the hypothesis (Chi-square test) that the data follow a binomial distribution. (8 marks)

- A survey of 320 families with 5 children each revealed the following distribution:

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable? (Tabulated $\chi^2_{0.05}$ for 5 degrees of freedom is 11.07) (4 marks)

- 4 coins are tossed together 160 times and following results are obtained:

No. of heads:	0	1	2	3	4
Frequency:	17	52	54	31	6

Under the assumption that coins are balanced, find the expected frequencies of getting 0, 1, 2, 3, 4 and test goodness of fit at 5% level. (Given $\chi^2_{0.05} = 9.488$) (7 marks)

- A die is thrown 60 times with the following results:

Force	1	2	3	4	5	6
Frequency	8	7	12	8	14	11

Test at 5% level if the die is unbiased, assuming that $\chi^2_{0.05}$ for 5 degrees of freedom is 11.1 (7 marks)