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December 2024

B.Tech. (IT/CSE/CE/CE (Hindi)/AI&ML)

(Third Semester)

Mathematics-III (Calculus and Ordinary Differential Equations) (BSC-301)

Time : 3 *Hours*] [*Maximum Marks* : 75

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) Find the upper and lower limits of the sequence $\left\{1, 2, \frac{1}{2}, 3, \frac{1}{3}, 4, \frac{1}{4}, 5, \frac{1}{5}, \ldots\ldots\right\}$. 1.5

(b) Define alternating series with an example. 1.5

(c) Test the convergence of the series :

$$\frac{5}{1.2.4} + \frac{7}{2.3.5} + \frac{9}{3.4.6} + \frac{11}{4.5.7} + \ldots$$ 1.5

(d) Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$. 1.5

(e) Find $f_x(x, y)$ and $f_y(x, y)$ when $f(x, y) = x \sin xy$. 1.5

(f) If $\varphi = 3x^2 z - y^2 z^3 + 4x^3 y + 2x - 3y - 5$, then find $\nabla^2 \varphi$. 1.5

(g) Evaluate the triple integral $\int_{y=0}^{1} \int_{z=0}^{1-y} \int_{x=0}^{2} dx\, dz\, dy$. 1.5

(h) State the order and degree of the following differential equation : 1.5

$$\frac{d^2 y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^3\right\}^{5/2}.$$

(i) Form the differential equation by eliminating the arbitrary constant k from : 1.5

$$y = kx + k - k^3.$$

(j) Find the Complementary Function (C.F.) for the following differential equation

$$\frac{d^2 y}{dx^2} + a^2 y = \tan ax.$$ 1.5

Part B

2. (a) Show that the sequence $\left\{\frac{n}{n^2+n-1}\right\}$ is decreasing and convergent. 7

(b) Find the Taylor's series expansion of $f(x) = \sin^2 x - x^2 e^{-x}$ about $x = 0$, up to the term containing x^4. 8

3. (a) Find the limit and test for continuity of the function : 7

$$f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x+y}, & x+y \neq 0 \\ 0, & x+y = 0 \end{cases}.$$

(b) Find the equations of the tangent plane and normal line to the surface $x^2 - 4y^2 + 3z^2 + 4 = 0$ at the point $(3, 2, 1)$. 8

4. (a) Evaluate

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)\,dx\,dy$$

by changing into polar coordinates. 7

(b) Using Green's theorem, evaluate $\int_C e^{-x}(\sin y\, dx + \cos y\, dy)$, C being the rectangle with vertices (0, 0), (π, 0), $\left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$. 8

5. (a) Solve the differential equation $\frac{dy}{dx} = \frac{y \sin 2x}{y^2 + \cos^2 x}$. 7

(b) Solve $p^2 y + 2px = y$, where $p = \frac{dy}{dx}$. 8

6. (a) Solve the following differential equation :

$$(x^2 D^2 - x D + 1)y = \left(\frac{\log x}{x}\right)^2.$$ 7

(b) Solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 \log x$ by the method of variation of parameters. 8

7. (a) Find the value of $J_{-\frac{1}{2}}(x)$, where $J_n(x)$ is Bessel's function of first kind of order n. 7

(b) Express the polynomial $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. 8