

# Asymptotic Notations

1. Big O ( $O$ )
2. Big Omega ( $\Omega$ )
3. Theta ( $\Theta$ )
4. Small ( $o$ )
5. Small Omega ( $\omega$ )

# Small 'o' Notation

Small o means **loose upper-bound** of  $f(n)$ . It is a rough estimate of the maximum order of growth whereas Big-O may be the actual order of growth.

In mathematical relation,

$f(n) = o(g(n))$  means

$\lim_{n \rightarrow \infty} f(n)/g(n) = 0$ ,  $f(n) = n^2 - 4 = o(n^3)$ ,

$n \rightarrow \infty$

$g(n) = n^3, (n^2 - 4)/n^3 = (1 - 4/n^2)/n = 1/\infty = 0$

# Small o Notation Examples

Examples:

Is  $7n + 8 \in o(n^2)$ ?  $/n^3/n^4/$

then check limits,

$$\lim_{n \rightarrow \infty} f(n)/g(n) = \lim_{n \rightarrow \infty} (7n + 8)/(n^2) = \lim_{n \rightarrow \infty} 7/n + 8/n^2 = 0$$

# Small Omega ( $\omega$ )

Small Omega means **loose lower-bound** of  $f(n)$ . It is a rough estimate of the minimum order of growth whereas Big-Omega may be the actual order of growth. In mathematical relation,

if  $f(n) \in \omega(g(n))$  then,

$$\lim_{n \rightarrow \infty} f(n)/g(n) = \infty$$

$$n \rightarrow \infty$$

# Small Omega ( $\omega$ ) Notation Examples

## Example:

1.  $4n + 6 \in \omega(1); Nx$  where  $x$  is less than the given power

here, we have functions  $f(n) = 4n + 6$  and  $g(n) = 1$

$$\lim_{n \rightarrow \infty} (4n + 6) / (1) = 4 * \infty + 6 = \infty$$

$$n \rightarrow \infty$$

2.  $5n^2 + 6n + 4 \in \omega(1)$  or  $\omega(n)$ ;

here, we have functions  $f(n) = 5n^2 + 6n + 4$  and  $g(n) = 1$

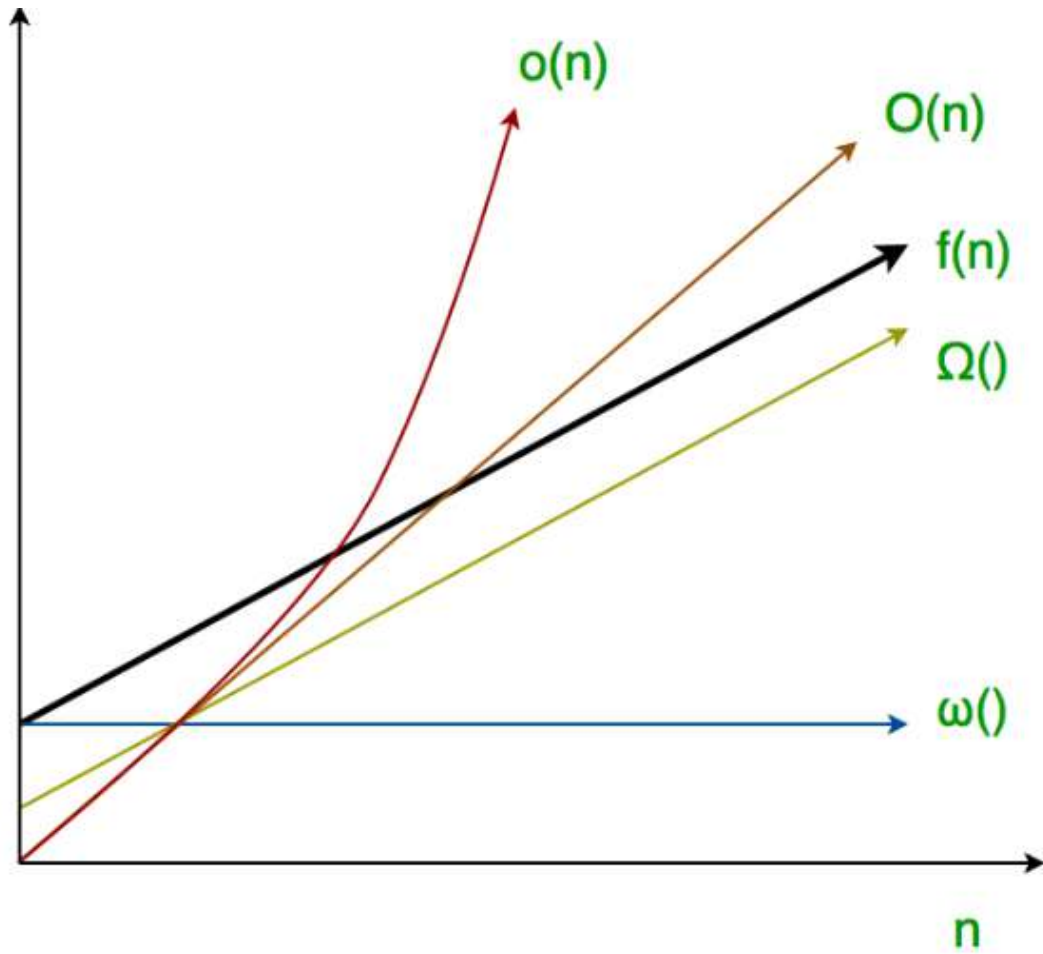
$$\lim_{n \rightarrow \infty} 5n^2 + 6n + 4 / (1) = \infty$$

$$n \rightarrow \infty$$

$$\lim_{n \rightarrow \infty} (5n^2 + 6n + 4) / (n) = \lim_{n \rightarrow \infty} 5n + 6 = \infty$$

$$n \rightarrow \infty$$

$$n \rightarrow \infty$$



# Linear Search

## Best Case

$$\Theta(1) / \Theta(c)$$

## Average Case

$$(n/2 - s) \leq c \leq (n/2 + s)$$

$$O(n)$$

## Worst Case

$$(n - 1) \leq c \leq (n + 1)$$

$\Theta(n)$  - when ur element is in the array

/  $\Omega(n)$  - not present

# Binary Search

## Best Case

$$\Theta(1)$$

## Average Case

$$\lg_2(n)/2 - 1 \leq c \leq \lg_2(n)/2 + 1$$

$$O(\lg_2 n)$$

## Worst Case

$$(\lg_2 n - 1) \leq c \leq (\lg_2 n + 1)$$

$$\Theta(\lg_2 n) / \Omega(\lg_2 n)$$

//that last element is the desired element