UNIT-1

Set :→

- A set is a collection of well defined and distinct objects.
- The objects of a set are called elements or points.
- Elements are denoted by lower case letters.
- A set is represented by a capital letter.
- If a is an element of a set S, then $a \in S$ (belongs to) Otherwise $a \notin S$ (does not belongs to.

Representation of set :→

⇒ Set can be represented in two ways :→

(i) Tabular or Roaster method.

(ii) Set Builder or Rule method.

☆ Roaster method :→

- A set is represented by listing the members within the { }.

e.g. (i) write the set of natural no. from 1 to 5.

⇒ $\{1, 2, 3, 4, 5\}$

(ii) write the set of letters of word INDIA

⇒ $\{I, N, D, A\}$

Builder / Rule method :→

• A set is represented by the property which the element satisfy.

eg (i) Set of Rational no. which are denoted by Q.

$\Rightarrow Q = \{x : x = \frac{p}{q}, p, q \in Z, q \neq 0\}$

(ii) write the given set in Set Builder form

$A = \{1, 3, 5, 7, 9, 11, 13\}$

$\Rightarrow A = \{x : x$ is an odd natural no. less than or equal to 13 $\}$

Q (i) write the set into tabular form :→

$A = \{x : 0 < x < 6, x$ is a set of integer $\}$

Ans $A = \{1, 2, 3, 4, 5\}$

(ii) $A = \{x : x^2 - 1 = 0, x$ is a natural no. $\}$

Ans $A = \{1\}$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x = 1, x = -1$ (discard)

$x = 1$

Note :→

- when the no. of elements in a set is small, then we use listing / Roaster method. But when the no. of elements is large or ∞, then we use set-Builder form.

Types of Set :→

1. Empty set :→

- A set having no element is called empty set / null / void set.
- It is denoted by { } or ϕ.

e.g. $A = \{x : x^2+1 = 0, x \in \mathbb{R}\}$

$A = \{x : 3 < x < 4, x \text{ is a whole no.}\}$

2. Singleton Set :→

- A set having only one element is called Singleton set.

$A = \{0\}, \{1\}, \{x\}$

3. Subset :→

- A set A is subset of set B, if each element of A is also an element of B.
- $A \subseteq B$

eg. $A = \{a, b, c\}$, $B = \{a, b, c, d\}$

$\Rightarrow A \subseteq B$ (A is Subset of B)

But B is not a Subset of A

$B \nsubseteq A$

4. Proper Subset :→

- If A is Subset of B and A is not equal to

B, then A is a proper subset of B.

- $A \subset B$, e.g. = $A = \{a, b, c\}$, $B = \{a, b, c, d\}$

$A = \{1, 6\}$, $B = \{1, 2, 3, 4, 5\}$

A is not a subset of B

- If A is subset of B, then B is called Superset of A.

$$\boxed{B \supseteq A}$$

e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 1\}$

$\Rightarrow B \supseteq A$

4. Equal Set :→

- Two set are equal if and only if $A \subseteq B$ and $B \subseteq A$ which means two sets are equal if and only if having same element.

e.g. $A = \{a, b\}$, $B = \{a, b\}$

$A = B$

5. Equivalent Set :→

- The set that contain same no. of elements although elements themselves may be different.

- Denoted by $A \sim B$.

e.g. $A = \{1, 2, 3\}$

$B = \{a, b, c\}$

$\Rightarrow A \sim B$

6. Comparable Set :→

- Two sets A and B are said to be comparable if A is a subset of B either B is subset of A.

e.g. (i) $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4\}$

$B \in A$

(ii) $A = \{a, b, c\}$, $B = \{$ all alphabets $\}$

$A \in B$

7. Universal Set :→

- A universal set is a set of collection of all elements of all related sets and subsets including itself.
- Different universal set are used in different content.
- The choice of universal set is not unique.
- It is denoted by U.

$A = \{1, 2, 3, 4\}$
$U = \mathbb{N}$
$U = \{1, 2 \ldots\}$

Theoram 1 :→ The empty set is subset of every set.

Theoram 2 :→

Every set is subset of itself.

Theoram 3 :→

$A \subseteq B$ and $B \subseteq A$

then $A = B$

Proof → $A \subseteq B$ let $x \in A$ be arbitrary.

If $x \in A$ then $x \in B \rightarrow (1)$

$B \subseteq A$ let $x \in B$, then $x \in A \rightarrow (2)$

from (1) and (2) then,

$x \in A$ and $x \in B$

$\Rightarrow \boxed{A = B}$

Theoram-4:→

If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

Proof → Let x be arbitrary element

Let $x \in A$ then $A \subseteq B$

then $x \in B$ → 1

$B \subseteq C$ then $x \in C$ → 2

$\Rightarrow \boxed{A \subseteq C}$

e.g. A = set of Natural no.
B = " whole no.
C = " Rational no.

$A \subseteq B$, $B \subseteq C$
then $A \subseteq C$

$\Rightarrow$ $\forall$ → for every, $\exists$ → There after.

Venn diagram:→

- It is a graphical representation of a set.
- This is used to indicate the relationship b/w sets.
- The universal set U is represented by a rectange and a subset A (& y) in interior of a circle.

(#) operations:→

1. Union:→

⇒ $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$ or $(\vee)$

- The union of two sets A and B written as $\boxed{A \cup B = \{ x : x \in A \text{ or } x \in B \}}$, where
  $A \cup B = x \in A$ or $x \in B$.

2. Intersection of Set:→ and $(\wedge)$

- The intersection of two set A and B written as $\boxed{A \cap B = \{ x : x \in A \text{ and } x \in B \}}$

3. Disjoint set:→

- Two set A and B are called Disjoint if they have no common element.

$$\boxed{A \cap B = \phi}$$

property 1:→

- Every element x in $A \cap B$ belong to both A and B. Hence $x \in A$ and $x \in B$. Thus,
  $A \cap B \subseteq A$ and of B.

$$\boxed{\begin{array}{l} A \cap B \subseteq A \\ A \cap B \subseteq B \end{array}}$$

Property 2:→

~~Every element $x \in A$~~

- Any element of x belongs to $A \cup B$, if $x \in A$ or $x \in B$. Hence of every element of A $\in A \cup B$ and every element in $B \in A \cup B$,

$$\boxed{\begin{array}{l} A \subseteq A\cup B \\ B \subseteq A\cup B \end{array}}$$

Theoram :→

For any set

(i) $A\cap B \subseteq A \subseteq A\cup B$

(ii) $A\cap B \subseteq B \subseteq A\cup B$

Theoram :→

~~A ∈ B iff A ∩ B = A~~

~~proof~~

The foll. are equivalent : $A\subseteq B$, $A\cap B = A$, $A\cup B = B$

Proof (i) Suppose $A\subseteq B$ and let $x\in A$ then $x\in B$,

Hence $x\in A\cap B$

$\Rightarrow A \subseteq A\cap B \rightarrow (1)$

$\Rightarrow$ By above theoram, $A\cap B \subseteq A \rightarrow (2)$

Therefore, $\boxed{A\cap B = A}$ [from (2) and (2)]

$\Rightarrow$ Conversely,

Suppose $A\cap B = A$ and let $x\in A$

then $x\in A\cap B$

Hence, $x \in A$ and $x \in B$

Therefore, $\boxed{A \subseteq B}$

Thus, $A \subseteq B$ is equivalent to $A \cap B = A$

(ii) Suppose $A \subseteq B$, let $x \in A \cup B$

then $x \in A$ or $x \in B$

If $x \in A$ then $x \in B$ because $A \subseteq B$

In either case, $x \in B$.

Therefore, $A \cup B \subseteq B$

By above theoram, $B \subseteq A \cup B$

Therefore, $\boxed{A \cup B = B}$

Conversely, Suppose $A \cup B = B$ and let $x \in A$

then $x \in A \cup B$ (by def^n of union of sets)

$\Rightarrow x \in B = A \cup B$

Therefore, $\boxed{A \subseteq B}$

Thus, both results show that $A \subseteq B$ is equivalent to $A \cup B = B$.

Thus, $A \subseteq B$, $A \cup B = A$ and $A \cup B = B$ are equivalent.

4. compliment of Set :→

- It is the set of element which belongs to U but does not belongs to A.

$\{x : x \in U \text{ and } x \notin A\}$

- $A^c = U - A$

5. Difference of set :→

- Difference of two set A and B written as A-B or A|B is the set of elements which belongs to A but do not belongs to B.

- $A|B = \{x : x \in A \text{ and } x \notin B\}$

6. Symmetric difference of set :→

- Notation = $\oplus$
- $(A-B) \cup (B-A)$
- $(A \cup B) - (A \cap B)$
- Symmetric difference of two set A & B written as $A \oplus B$, contains elements which are in A or B but not in both.

Theoram :→ If A and B are two sets, then $A \oplus B = (A \cup B) \setminus (A \cap B)$

$A \oplus B = (A-B) \cup (B-A)$

proof →

~~Proof~~

Let $x \in A \oplus B$

then

$\Leftrightarrow x \in (A-B) \cup (B-A)$

$\Leftrightarrow x \in A-B$ or $x \in B-A$

$\Leftrightarrow (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \notin A)$

~~$\Leftrightarrow (x \in B$ or $(x \in A$ and $x \in B)$ and $x \in A$ or $(x \in A$ and $x \in B)$~~

$\Leftrightarrow [x \in A$ and $x \notin B]$ or $x \in B$ and $[x \in A$ or $x \notin B$ or $x \notin A]$

$\Leftrightarrow [(x \in B$ or $x \in A)$ and $(x \in B$ or $x \notin B)$ and

$[(x \notin A$ or $x \in A)$ and $(x \notin A$ or $x \notin B)]$

$\Leftrightarrow [x \in A \cup B$ and $x \in U]$ and $[x \in U$ and $x \notin A \cap B]$

$\Leftrightarrow x \in A \cup B$ and $x \notin A \cap B$

$\Leftrightarrow x \in (A \cup B) | (A \cap B)$

i.e. $A \oplus B = (A \cup B) - (A \cap B)$.

e.g. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4\}$

$B = \{5, 6, 7, 8\}$, $C = \{9, 10\}$, $D = \{1, 3, 5, 7, 9\}$

$\Rightarrow$ (i) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8)$

(ii) $A \oplus B = \{1, 2, 3, 4, 5, 6, 7, 8)$

(iii) $C - D = 10$

(iv) $A \cap \phi = \phi$

(v) $(C \cup B)' = U - (C \cup B) = \{2, 4, 6, 8\}$

Set

finite
e.g (0 --- 9)

Infinite

Countably infinite
(e.g) - IN, Odd no. set

uncountable infinite.
(e.g : Real no.

Countable set

finite

Countably infinite.

finite set :→

- A set is said to be infinite if S is empty or S contains m elements where m is +ve integer.

e.g =(i) {0 --- 9},
(ii) {a, e, i, o, u}

Infinite Set :→

A set which neither a null set or a finite set. is called infinte set.
g. set of natural no.'s, N={1, 2 --- }

counting element in finite set :→

- The notation $n(A)$ or $|A|$ [Cardinality of A] will denoted the no. of elements in a set A.

Power Set :→

- The set of all possible subsets of a given set A.
- It is denoted by $P(A)$.

e.g. $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\},$
$\{b, c\}, \{a, c\}, \{a, b, c\}\}.$

Note → If a Set S contains n elements then power set of S contains 2^n element.

Fundamental product :→

⇒ A fundamental product of Sets $A_1, A_2, --- A_n$ is an expression of the form $A_1^* \cap A_2^* -- A_n^*$ where A_i^* is either A_i or A_i^c

e.g. A_1, A_2, A_3

$P_1 = A_1 \cap A_2 \cap A_3$

$P_2 = A_1 \cap A_2 \cap A_3^c$

$P_3 = A_1 \cap A_2^c \cap A_3$

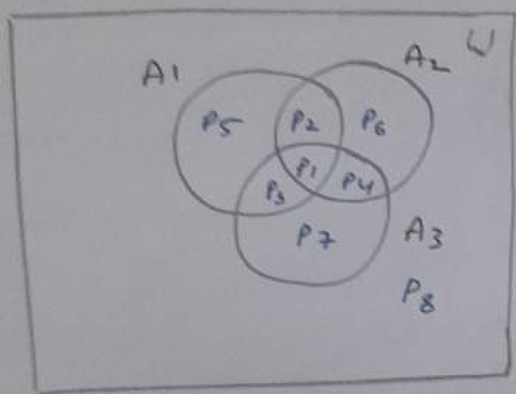
$P_4 = A_1^c \cap A_2 \cap A_3$

$P_5 = A_1 \cap A_2^c \cap A_3^c$

$P_6 = A_1^c \cap A_2^c \cap A_3$

$P_7 = A_1^c \cap A_2 \cap A_3^c$

$P_8 = A_1^c \cap A_2^c \cap A_3^c$



Note :-

(i) For N sets, we may have 2^N fundamental product.

(ii) Any two such fundamental sets are disjoint.

(iii) The universal set 'U' is the union of all fundamental product.

Algebra of Sets:→

- Sets under the operation of union intersection and complementary satisfy various loss.

Laws of Sets:→

(1) Identity Law:→

(a) $A \cup \phi = A$

(b) $A \cap U = A$

(2) Domination Law.

(a) $A \cup U = U$

(b) $A \cap \phi = \phi$

3. Idompotent Law:-

(a) $A \cup A = A$

(b) $A \cap A = A$

4. Complement Law:-

(a) $A \cup A^c = U$

(b) $A \cap A^c = \phi$

5. (a) $U^c = \phi$

(b) $\phi^c = U$

6. Associative Law:-

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

7. Commutative Law:-

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$

8. Distributive Law:-

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = A (\cap B) \cup (A \cap C)$

9. Involution law:-

- $(A^c)^c = A$

e.g. Let $U = \{1, 2, \cdots 10\}$

$A = \{1, 2, 3, 4, 5\}$

$A^c = \{6, 7, 8, 9, 10\}$

$(A^c)^c = \{1, 2, 3, 4, 5\}$

10: De Morgans Law:→

(a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

Proof (a):→ $(A \cup B)^c = A^c \cap B^c$

T.P = $(A \cup B)^c \subseteq A^c \cap B^c$

$A^c \cap B^c \subseteq (A \cup B)^c$

$\Rightarrow$ Let $x \in (A \cup B)^c$

$x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

$\Rightarrow (A \cup B)^c \subseteq A^c \cap B^c$

$\Rightarrow$ Let $x \in A^c \cap B^c$ be arbitrary

$x \in A^c$ and $x \in B^c$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)^c$

$\Rightarrow (A \cup B)^c = \{x : x \in (A \cup B)^c\}$

$= \{x : x \in A \cup B\}$

$= \{x : x \notin A$ and $x \notin B\}$

$= \{x : x \in A^c$ and $x \in B^c\}$

$= \{x : x \in A^c \cap B^c\}$

$= A^c \cap B^c$

(a) T.P $\Rightarrow (A \cup B)^c = A^c \cap B^c$

<u>Proof</u> $(A \cup B)^c = \{x \mid x \notin A \cup B\}$

$= \{x \mid \neg x \in A \cup B\}$

$= \{x \mid \neg (x \in A \text{ or } x \in B)\}$

$\Rightarrow x \mid x \notin A \text{ and } x \notin B\}$

$\Rightarrow x \mid x \in A^c \text{ and } x \in B^c\}$

$\Rightarrow x \mid x \in A^c \cap B^c$

$\Rightarrow A^c \cap B^c$

$\neg$ means (not)

(b) T.P $= (A \cap B)^c = A^c \cup B^c$

<u>Proof</u> $(A \cap B)^c = \{x \mid x \notin A \cap B\}$

$= \{x \mid \neg x \in (A \cap B)\}$

$= \{x \mid \neg (x \in A \text{ and } x \in B)\}$

$= \{x \mid x \notin A \text{ or } x \notin B\}$

$\Rightarrow \{x \mid x \in A^c \text{ or } x \in B^c\}$

$\Rightarrow \{x \mid x \in A^c \cup B^c\}$

$\Rightarrow A^c \cup B^c$

Duality :→

- The dual of an eq^n involving set is obtain by interchanging union and intersection and universal & empty set.
- Each law of algebra imply on its all dual. It is the fact of set Algebra called the principal of Duality.
- It is denoted by E^*.

e.g. $(U \cap A) \cup (B \cap A) = A$

dual $(\phi \cup A) \cap (B \cup A)$

$A \cap (B \cup A)$

A

- If an eq^n E is an identity then its dual is also an identity.

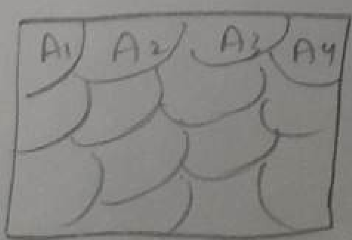
Partition :→

- A partition of S is a collection of sets $\{A_i\}$ of non-empty subsets S, such that :-

(a) Each A in a S belongs to one of the A_is.

(b) The set A_i are mutually disjoint, i.e if $A_j \neq A_K$, then $A_j \cap A_K = \phi$.

e.g.



Consider the foll. of collection of subset of S such 1 to 9 element.

(i) $\{\{1,3,5\}, \{2,6\}, \{4,6,9\}\}$

(Not Partition).

(ii) $\mathcal{F} = \{\{1,3,5\}, \{2,4,6,8\}, \{5,7,9\}\}$
$\Rightarrow$ {Partition}

(iii) $= \{\{1,3,5\}, \{2,4,6,8\}, \{7,9\}\}$
$\Rightarrow$ (Partition)

Generalised Set operations :→

- Consider first a finite no. of set $A_1, A_2 \ldots A_m$ then the union and intersection of these are denoted and defined respectively by :→

(i) $A_1 \cup A_2 \cdots \cup A_m$

$\Rightarrow \bigcup_{i=1}^{m} A_i = \{x : x \in A_i, \text{ for some } A_i\}$

(ii) $A_1 \cap A_2 \cdots \cap A_m$

$\Rightarrow \bigcap_{i=1}^{m} A_i = \{x : x \in A_i \text{ for every } A_i\}$

- Now let this A is a collection of sets then the union and intersection of the sets in the collected is denoted and defined resp. by :-

$\Rightarrow A = A_1, A_2 \cdots A_m, \cdots$

(i) $\cup (A_i : A_i \in A\} = \{x : x \in A_i, \text{ for some } A_i \in A\}$

(ii) $\cap (A_i : A_i \in A\} = \{x : x \in A_i, \text{ for every } A_i \in A\}$

e.g. $A_1 = \mathbb{N}$

$A_2 = \{2, 3, 4 \cdots\}$

$A_3 = \{3, 4 \cdots\}$

$\vdots$

$A_n = \{n, n+1 \cdots\}$

find (i) $\cup \{A_K : K \in \mathbb{N}\}$

(ii) $\cap \{A_K : K \in \mathbb{N}\}$

Soln (i) $\cup \{A_K \mid K \in \mathbb{N}\}$

$\Rightarrow A_1 \cup A_2 \cup A_3 \cdots \cup A_n$

$\Rightarrow \mathbb{N}$

(ii) $\cap (A_K : K \in \mathbb{N}\}$

$\Rightarrow A_1 \cap A_2 \cap A_3 \cdots \cap A_n$

$\Rightarrow \phi$ (Empty Set).

Cartesian Product :-

- Let A and B be two sets then the Cartesian production of A and B is denoted $A \times B$ is the set of all ordered pairs of the form a, b where $a \in A$ and $b \in B$. 2 tupple element.

- $A \times B = \{(a, b) , a \in A, b \in B\}$

e.g. $A = \{1, 2\}$, $B = \{a, b, c\}$

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$

$B \times A = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

- $n(A) = 2$, $n(B) = 3$, $n(A) \cdot n(B) = 6$,
  $n(A \times B) = 6$.

Note (i) $A \times B \neq B \times A$
(ii) $A^2 = A \times A$

Note , Let $A_1, A_2 \ldots A_n$ be finite no. of sets then the set of all ordered n tupples $(a_1, a_2 \ldots a_n)$ where $a_1 \in A_1$, $a_2 \in A_2$ and $\ldots$ $a_n \in A_n$, is called Product of sets $A_1, A_2 \ldots A_n$ is denoted by (i) $A_1 \times A_2 \times \ldots \times A_n$

or $\prod_{i=1}^{n} A_i$

$\boxed{n(A \times B) = n(B \times A)}$

e.g. $A = \{1, 2, \ldots 6\}$
$B = \{x, y, z\}$

$n(A) = 6$, $n(B) = 3$

$n(A \times B) = 18$

Relations :→

- Let A and B be sets, ~~are~~ a binary Relation or Simply Relation are from A to B is a Subset of $A \times B$.

- Given $a \in A$ and $b \in B$, we write.

→ a related to B ($a R B$) if (a,b) ordered ~~pair~~ pair belongs to R.

→ $a \not R b$ if (a,b) ordered pair does not belongs to R.

e.g. $A = \{1, 2\}$, $B\{x, y, z\}$

$R = \{(1,x), (1,y), (2,z)\}$

$\Rightarrow 1 R x, \quad 1 R z, \quad 1 \not R y.$

$2 \not R x, \quad 2 \not R y, \quad 2 R z$

Note If R is a Relation from a set to itself i.e. $R \subseteq A \times A$, then we say R is Relation on A.

$R \subset A \times A$

Domain & Range :-

Domain of a relation R is the set of all first element of the ordered pair which belongs to R and the Range is the set of 2^{nd} elements.

e.g. Let $A = \{1, 2, 3\}$, $B = \{x, y, z\}$

$R = \{(1, y), (1, z), (3, y)\}$

Domain $= \{1, 3\}$, Range $= \{y, z\}$

Inverse Relation :-

$\rightarrow R \longrightarrow A \rightarrow B = \{(a, b) : a \in A, b \in B\}$

$\rightarrow R^{-1} \rightarrow B \rightarrow A = \{(b, a) : (a, b) \in R\}$ $b \in B, a \in A$

- Let R be any relation from a set A to B, then the inverse of R is denoted R^{-1}, is the relation from B to A, which consist of those ordered pair which when reversed belong to R.

e.g. $A = \{1, 2\}$, $B = \{a, b, c\}$

$R = \{(1, a), (2, c), (1, b)\}$

$R^{-1} = \{(a, 1), (c, 2), (b, 1)\}$

Pictorial Representation of Relations :-

(i) Representation by Matrix

Let M and N be the no. of elements in set A and B resp., then a Relation R from A to B can be represented by a $M \times N$ matrix.

- $M = [m_{ij}]$
- $m_{ij} = \begin{cases} 0 \; ; & (a,b) \notin R \\ 1 \; ; & (a,b) \in R \end{cases}$

e.g. (i) $A = \{1, 2, 3\}$, $B = \{x, y, z\}$

$R = \{(1,y), (1,z), (3,y)\}$

matrix Representation

$$\begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(ii) $S = \{(a,1), (b,2), (c,3)\}$,

$A = \{1, 2, 3\}$, $B = \{a, b, c\}$

$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Identity matrix

Pictorial Representative of Relations :-

(i) Representation by. Matrix

Let M and N be the no. of elements in Set A and B resp., then a Relation R from A to B can be represented by a $M \times N$ matrix.

- $M = [M_{ij}]$
- $M_{ij} = \begin{cases} 0 ; & (a,b) \notin R \\ 1 ; & (a,b) \in R \end{cases}$

e.g. (i) $A = \{1, 2, 3\}$, $B = \{x, y, z\}$

$R = \{(1,y), (1,z), (3,y)\}$

Matrix Representation

$$\begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(ii) $S = \{(a,1), (b,2), (c,3)\}$,

$A = \{1, 2, 3\}$, $B = \{a, b, c\}$

$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

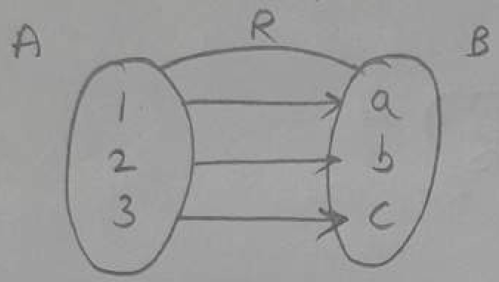
Identity matrix

(ii) <u>Representation by Arrow Diagram</u>:→

The elements of Set A and elements of Set are written in two disjoint disc, then draw an arrow from $a \in A$ to $b \in B$ whenever aRb

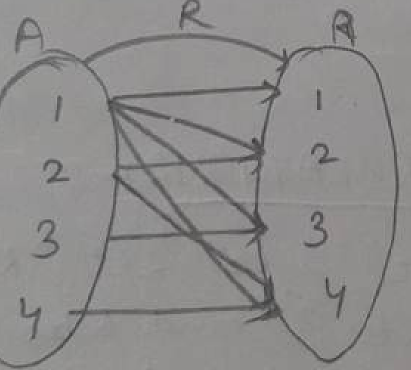
<u>e.g.</u> (i) $A = \{1, 2, 3\}$, $B = \{a, b, c\}$

$R = \{(1, a), (2, b), (3, c)\}$



(ii) Let $A = \{1, 2, 3, 4\}$ and R be the Relation on A defind by a divides B

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$



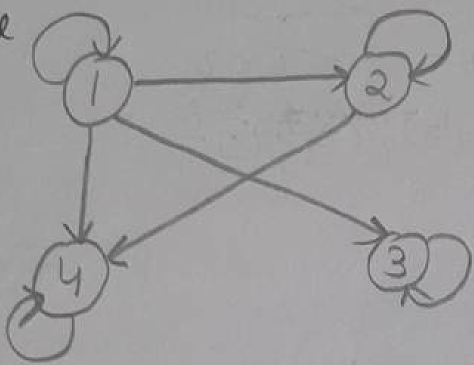
(iii) <u>Directed Graph on Relation Of Set</u>:→

- write down the element of set and then we drawn an array to each element of x to each element of y whenever x is related to y. This diagram is called Directed Graph of Relation.

e.g. (i) $A = \{1, 2, 3, 4\}$, $R = a/b$

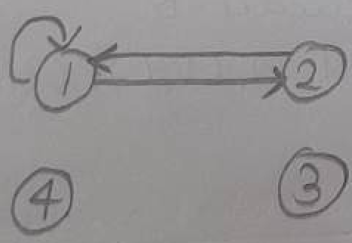
$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$.

Directed Graph.



(ii) $A = \{1, 2, 3, 4\}$,
$R = \{(1,1), (1,2), (2,1)\}$

Directed Graph.

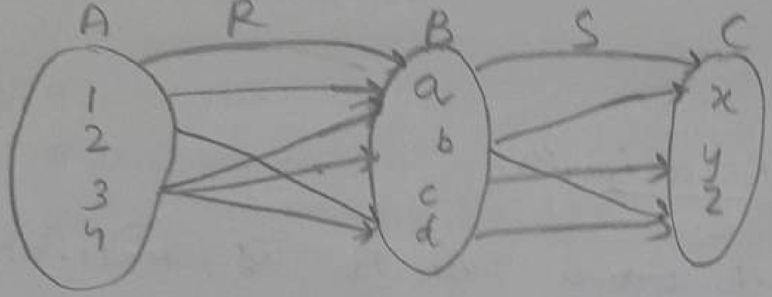


COMPOSITION OF RELATION :-

- Let A, B, C be sets and let R be a relation from A to B and S be a relation from B to C. i.e $R \subseteq A \times B$ and $S \subseteq B \times C$, then R and S give rise to a Relation from A to C denoted by $R \circ S$ / RS and defined by

$a\,(R \circ S)\,c$ for some $b \in B$ when we have, $a R b$, $b S c$.

e.g. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$
$C = \{x, y, z\}$
$R = \{(1,a), (2,d), (3,a), (3,b), (3,d)\}$
$S = \{(b,x), (c,y), (d,z)\}$



$\Rightarrow$ a (ROS) C = (i) 2 (ROS) z for some $d \in B$; we have 2 R d and d S z

(ii) 3 (ROS) x for some $b \in B$, we have 3 R b and b S x

(iii) 3 (ROS) z for some $b \in B$, we have 3 R B and b S z.

Types of Relation:→

1. Identity Relation:→

- A relation R in a set A, is said to be Identity Relation if $R = \{(x,x) ; x \in A\}$. This is also called Diagonal Relation.

e.g. $A = \{1, 2, 3, 4\}$
$R = \{(1,1), (2,2), (3,3), (4,4)\}$

2. Universal Relation:→

- A relation R in set A, is said to be universal Relation, if $R = A \times A$.

e.g. if $A = \{x, y, z\}$

$R = \{(x,x), (x,y), (x,z), (y,y), (y,x), (y,z), (z,z), (z,x), (z,y)\}$

(iii) Reflexive Relation :→

- A relation R on a Set A is Reflexive, if aRa for every $a \in A$ i.e. ordered pair $(a,a) \in R$ for $a \in A$.

e.g. Let $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1), (2,2), (3,3), (4,4), (1,2)\}$.

Note • Empty Set is not Reflexive.

(iv) Symmetric Relation :→

- A relation R on a set A is Symmetric, if whenever aRb then bRa i.e whenever $(a,b) \in R$ then $(b,a) \in R$.

e.g. Let $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$.

(V) Anti-Symmetric Relation :→

A relation R on a Set A is Anti-Symmetric, if whenever aRb and bRa then $a=b$, i.e if $a \neq b$ and whenever aRb then $b \not R a$.

e.g. $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (2,2), (3,3), (4,4)\}$ → Anti - Symmetric

$R = \{(1,1), (2,2), (3,3), (1,2)\}$ → Anti - Symmetric.

e.g. (i) Let R be a relation of divisiblity on a Set of +ve integers.

$A = Z^+$

$R = \{(1,1), (2,2), (3,3) \ldots$
$(1,2), (1,3) \ldots$
$(2,4), (2,6) \ldots\}$

This is anti - Symmetric Relation.

(ii) R is defined in less than or equal to on the set of integers. $\boxed{a \leq b}$

$R = \{(1,1), (1,2), (1,3) \ldots$
$(2,2), (2,3) \ldots$
$(3,3), \ldots\}$

This is Anti - Symmetric Relation.

(VI) Transitive Relation :→

- A relation R on a set A is transitive, if aRb, bRC then aRC i.e, whenever (a, b) & $(b, C) \in A$ then $(a, c) \in A$.

e.g. $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$

- A relation of perpendicularity is not Transitive.

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eg = • Set | Relation of parallel lines

⇒ This relation is Symmetric, Reflexive and Transitive.

• perpendicular lines → Relation is Symmetric

Note: if a relation is Reflexive, Symmetric & Transitive, then the relation is called equivalence relation.

• Equivalence Relation:-

⇒ Consider a non-empty set A then a relation R is an equivalence relation, if R is reflexive, Symmetric & Transitive

eg (1) let R be a relation on a set N of the integer defined by $R = \{(a,b) : a+b \text{ is even}\}$

⇒ $A = Z^+$

(i) Reflexive

aRa

$a+a$ is even

(ii) Symmetric

let aRb

⇒ $a+b$ is even

$b+a$ is even

⇒ bRa

(iii) Transitive

let aRb and bRc

$a+b$ is even and $b+c$ is even

(i) a, b, c even ⇒ $a+c$ is even

(ii) $a, b, c \to$ odd ⇒ $a+c$ is even

⇒ aRc

(ii) Let R be a relation on the set of integers defined by $R = \{(x,y) : x-y$ is divisible by $6\}$

Prove that R is equivalence relation.

$\Rightarrow$ $A = Z$

$R = \{(x,y) = x-y = 6n\}$

(i) Reflexive:- xRx

$x - x = 6n$

(ii) Symmetric xRy

$x-y = 6n$

$-(y-x) = 6n$

$\Rightarrow yRx$

(iii) Transitive

let xRy and yRz

$x-y = 6n$ and $y-z = 6m$; $m, n \in N$

$x-z = 6n + 6m$

$x - z = 6t$

$\Rightarrow xRz$

Partial Order relation :-

• A relation ~~are~~ R on a set ~~S~~ S is ~~called~~ said to be Partial Order relation if R is ~~Symmetric~~, Reflexive anti-Symmetric and Transitive.

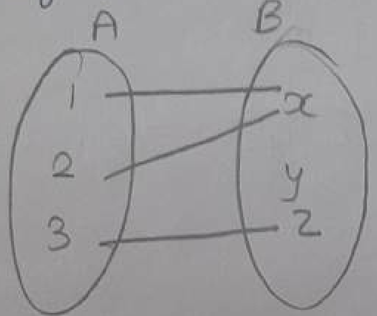
e.g.

Functions:→

- A binary relation R → A to B is

- If an element $a \in A$ then $f(a)$ denotes the unique element b of B set which f assigns to a, i.e $f(a)=b$. It is called the image under f and a is called pre-image of B.

- The set of all images under f is called the Range of f denoted by Ran(f) or f(A).

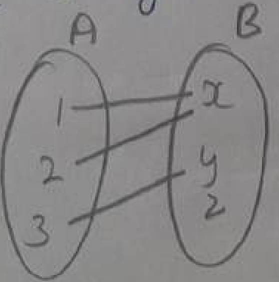


Types of functions:→

(i) Many-one func^n:→

The func^n f defined from A to B is many-one, if two or more elements in A have same image under f.

e.g.

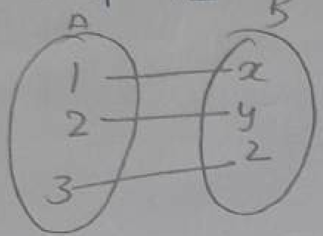


(ii) One - one fuc^n:- | Injective

- A fuc^n $f \to A$ to B is said to be one to one, if different elements in the domain A have distinct images.

i.e $f(a_1) = f(a_2)$

$\Rightarrow a_1 = a_2$



e.g. Show that fuc^n $f \to R$ to R given by $f(x) = x^3 + x$ is one - one

Sol^n Let $f(x) = f(y)$ $[x^3 - y^3 = (x-y)(x^2+y^2+xy)]$

$x^3 + x = y^3 + y$

$(x^3 - y^3) + (x - y) = 0$

$(x-y)(x^2+y^2+xy) + (x-y) = 0$

$x - y\,(x^2 + y^2 + xy + 1) = 0$

$x - y = 0$

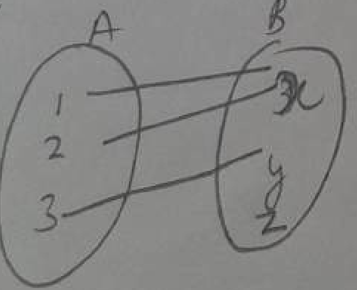
$\Rightarrow x = y$

Hence F is one - one fuc^n

(iii) Into fuc^n:→

- A fuc^n $F : A \to B$ is into fuc^n, if F is a proper subset of $f(A) \subset B$ and $f(A) \neq B$ i.e, there is atleast one element of B which is not an image of an element of A.

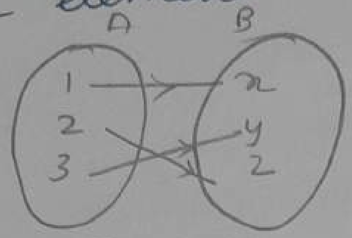
e.g.



(iv) Onto fucⁿ :- (Surjective)

- A fucⁿ $f: A \mapsto B$ is said to be onto fucⁿ, if each element of B is the image of some element A.

$\boxed{f(x) = y}$



$f(A) = B$

(v) Bijective fucⁿ :- (Invertible)

- A fucⁿ $F: A$ to A is ~~said to~~ one to one correspondence bijective if f is both one ~~&~~ one and onto.

e.g. $f(x) = x$

e.g. $f(x) = 4x + 3$

one-one, $f(x) = f(y)$

$$4x + 3 = 4y + 3$$

$$4x = 4y$$

$$\Rightarrow x = y.$$

onto, $f(x) = y$

$$4x + 3 = y$$

$$x = \frac{y-3}{4}$$

Q. Let $A = R - \{3\}$ and $B = R - \{1\}$ consider the fucn $f: A$ to B defined by $f(x) = \frac{x-2}{x-1}$.
Show that the given fucn is both one-one & onto.

Solⁿ $f(x) = \frac{x-2}{x-1}$

$\Rightarrow f(x) = f(y)$

$\Rightarrow \frac{x-2}{x-1} = \frac{y-2}{y-1}$

$\Rightarrow (x-2)(y-1) = (y-2)(x-1)$

$\cancel{xy} - x - 2y + \cancel{2} = \cancel{yx} - y - 2x + \cancel{2}$

$+x = +y$

$x = y$

Hence, f is one-one

$\Rightarrow f(x) = y$

$\frac{x-2}{x-1} = y$

$xy - y = x - 2$

$xy - x = y - 2$

$x(y-1) = y - 2$

$x = \frac{y-2}{y-1}$

Hence, f is onto.

Inverse of fuc^n :-

- Let f be a one-one correspondance from A set to B. The inverse fuc^n F is the fuc^n that assigns to each b element belonging to B set. The unique element $a \in A$. such that $f(a)=b$. The inverse of a fuc^n is denoted by f^{-1} Hence $f^{-1}(b)=a$ when $f(a)=f(b)$.

e.g. Show that the mapping $f: R$ to R is defined by $f(x)= 4x-1$ define its inverse.

$\Rightarrow$ one-one

$$f(x) = f(y)$$
$$4x-1 = 4y-1$$
$$4x = 4y$$
$$\Rightarrow x=y$$

Hence F is one-one.

$\Rightarrow$ onto

$$f(x) = y$$
$$4x-1 = y$$
$$x = \frac{y+1}{4}$$

Hence, f is onto.

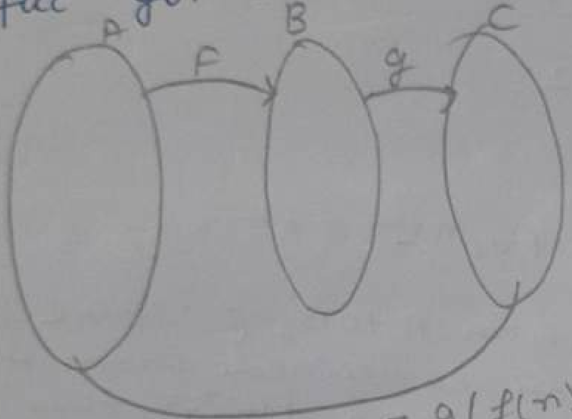
$\Rightarrow$ Inverse $f^{-1}(y)=x$ when $f(x)=y$

$$f^{-1}(y) = \frac{y+1}{4}$$

Composition of fuc^n:→

- Let F defined from X to Y and g defined from Y to Z with the property that the range of F is subset of the domain of g. define a new fuc^n gof from X to Z as follows:→

The fuc^n gof is called composition of f & g.



$$gof(x) = g(f(x))$$

e.g. (i) $F(x) = x^2$ and $g(x) = x+5$ where $x \in$ Set of Real no.

Find comp. fog & gof.

Sol^n $F: R \to R$, $g: R \to R$

$f(x) = x^2$, $g(x) = x+5$

$(fog)(x) = f(g(x))$

$\Rightarrow f(x+5) = (x+5)^2$

$(goF)(x) = g(f(x))$

$= g(x^2) = x^2+5$

(ii) Let f and g be real value fuc^n defined $f(x) = \sin x$ & $g(x) = x^2$, where $x \in R$. Find fog & gof.

Sol^n $f(x) = \sin x, \quad g(x) = x^2$

$$fog = f(g(x))$$
$$= f(x^2)$$
$$= \sin x^2$$

$$gof = g(f(x))$$
$$= g(\sin x)$$
$$gof = (\sin x)^2$$

Association of Fuc^n:-

- If f is defined from $A \to B$, $g : B \to C$; $h : C \to D$ are 3 fuc^n then gof is defined from $A \to C$ and $hog : B \to D$ are also fuc^{n}s then we can form $(hog)of : A \to D$ and $ho(goF)$: $A \to D$, assuming that $a \in A$ we have

$$\Rightarrow (hog)oF = ho(goF)$$
$$\Rightarrow ((hog)of)(x)$$
$$\Rightarrow (hog)f(x)$$
$$\Rightarrow ho(g(f(x)))$$
$$\Rightarrow (ho(goF))(x)$$

e.g. Let $f(x) = x+2$, $g(x) = x-2$ and $h(x) = 3x$. Find gof, hog, fog, $(hog)oF$, $ho(goF)$

Sol^n
$$goF = g(F(x))$$
$$goF = g(x+2)$$
$$goF = x$$

$\Rightarrow Fog = F(g(x)$

$Fog = F(x-2)$

$\Rightarrow Fog = x$

$\Rightarrow hog = h(g(x)$

$hog = h(x-2)$

$hog = 3(x-2)$

$\Rightarrow ho(goF) = h(x)$

$= 3x$

$\Rightarrow$ ~~holgo~~ $(hog)oF = (3x-2)^{F}$

$\Rightarrow 3x-2+2$

$\Rightarrow 3x$

Identity Function :→

- If x is any set and the mapping $F: A \to A$ is defined by $F(a) = A$ i.e every element of the set A is image of itself then the mapping is called Identity fucn.
- It is always bijective mapping.
- Denoted by I(A).

constant function :→

A mapping in which every element of the domain is assigned to same element of the codomain is called a constant mapping i.e the range of the

Constant mapping is a set which contains only one element. Thus, the mapping $f: X \to Y$ is a constant func^n if $f(x) = K$ where $K \in Y$. $x \in X$ such that

constant

e.g. $F: R \to R$ as $F(x) = 3$

* $F \circ F^{-1} = I$

$(F \circ f^{-1})(x) = I(x)$, $f(x) = x+3$

$f(f^{-1}(x)) = x$

$f^{-1}(x) + 3 = 3$

$f^{-1}(x) = x - 3$

Ternary Relation :→

- A ternary relation is a set of ordered triple in particular, if S is any set, then the set $S \times S \times S$ is called ternary relation on S.

Ir-Reflexive Relation :-

- A relation on a set A is said to be ir-reflexive if $(a, a) \notin R$ for every $a \in A$.

Principle of Mathematical Induction :

- Let P be a proposition defined on the +ve integers n i.e P(n) is either true or false for each $n \in N$ (natural no set). Suppose P has the following two properties.

(i) P(1) is true.

(ii) P(K+1) is true whenever P(K) is true for every +ve integer i.e P is true for every $n \in N$.

e.g: P(n) = $1+2+3 - - - n = \frac{n(n+1)}{2}$

Show that by Mathematical induction

(i) for $n=1$, $P(1) = 1 = \frac{1\times \cancel{2}}{\cancel{2}} = 1$

$\Rightarrow P(1)$ is true

(ii) let P(K) is true for some $K \in N$

i.e $1+2+ - - - +K = \frac{K(K+1)}{2}$

(iii) consider, $P(K+1) = 1+2+ - - - +K+1$

$= (1+2+ - - +K) + K+1$

$= \frac{K(K+1)}{2} + K+1$

$= K+1\left[\frac{K}{2}+1\right]$

$\Rightarrow \frac{(K+1)(K+2)}{2}$

(ii) Show that $1+3+5 - - - 2n-1$ is equal to n^2 for all integers k greater than equal to 1.

Soln (i) For $n=1$, $P(1) = 1 = 1^2 = 1$

$\Rightarrow P(1)$ is true

(ii) let $P(K)$ is true for some $K \in N$

i.e. $1+3+5+ - - 2K-1 = K^2$

(iii) consider, $P(K+1) = 1+3+5 - - - [2(K+1)-1]$

$= 1+3+5 - - [2K+2-1]$

$= 1+3+5+ - - - (2K+1)]$

$\Rightarrow 1+3+5 - - + (2K-1) + 2K+1$

$\Rightarrow K^2+2K+1$

$= (K+1)^2$

(iii) Prove that $1^2+2^2+ - - - n^2 = \frac{n(n+1)(2n+1)}{6}$

Soln (i) for $n=1$, $P(1) = 1 = \frac{1(2)(3)}{6} = \frac{\cancel{6}}{\cancel{6}} = 1$

$\Rightarrow P(1)$ is true

(ii) Let $P(K)$ is true for some $K \in N$.

i.e. $1^2+2^2+ - - - K^2 = \frac{K(K+1)(2K+1)}{6}$

(iii) consider $P(K+1) = 1^2+2^2 - - - (K+1)^2$

$\Rightarrow 1^2+2^2 - - - - + K^2 + (K+1)^2$

$\Rightarrow \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$\Rightarrow (K+1)\left[\frac{K(2K+1)}{6} + K+1\right]$

$\Rightarrow (K+1)\left[\frac{2K^2+K+6K+6}{6}\right]$

$$\Rightarrow (K+1)\left[\frac{2K^2+7K+6}{6}\right]$$

Division Algorithm:→

let ~~a~~ d be an integer ~~and~~ then their unique integers q and ~~r~~, with greater than equal to zero such that $a = dq + r$.

GCD :→

- Greatest Common Divisor
- let a and b be integers where both are not equal to zero then the largest integers d such that d divides a and d divides b, is called the greatest common divisor of a and b.
- It is denoted by GCD(a, b).

Euclid's division Lemma

- Given the integers a and b there exist unique integers Q and R satisfying $a = bq + r$, where $r \geq 0$ but less than b.

⇒ Euclid's division algorithm:→

- To obtain gcd of two +ve integers say C and D follow the steps below.

⇒ Let $C > D$

(i) Apply Euclid's division Lemma to C and D, So, we find whole no's Q and R such that $C = DQ + R$, where $R \geq 0$ but $R < d$

(ii) If $R=0$ then D is the GCD of C and D.

(iii) if $R \neq 0$, Apply the division Lemma to b and r.

e.g. gcd(24, 36)

$36 = 24 \times 1 + 12$

$24 = 12 \times 2 + 0$

$\boxed{gcd = 12}$

Q. use Euclid's Algo. to find gcd of 91 & 287

$287 = 91 \times 3 + 14$

$91 = 14 \times 6 + 7$

$14 = 7 \times 2 + 0$

$\Rightarrow gcd = 7$

Fundamental Theoram of Arithmetic :-

- Every composite no. can be expressed or factorised as a product of primes and this factorisation is unique. Apart from the order in which the prime factor occurs.

Well Ordered Sets :-

- A Set with some order is called well ordered if any non empty Subset has a least element.

well ordering Principle :- (WOP)

- Every non-empty Subset of N ie natural no. has a smallest element.

⇒ for positive integers (WOP)

- Let S be a set of +ve integers containing one or more integers all of which are greater than some fixed integer. Then S has a least element.

Note The principle of well ordering is an existance theoram. It does not tell us which element is the smallest nor does it tell how to find out the smallest element.

Recurrence Relation :-

- A recurrence relation for a sequence a_0, a_1, a_2 ------ is a formula that relates each term a_k to certain of its preecedors a_{k-1}, a_{k-2} --- a_{k-i}, where i is an integer with $k-i \geq 0$, the initial conditions for such a recurrence relation specify the values a_0, a_1, a_2 --- a_{i-1}, if i is a fixed integer or a_0 a_1 --- a_m where m is integer, greater than equal to zero if i depends on k.

e.g. Define a sequence c_0, c_1, c_2 ---- recursively as follows:-

(i) $C_k = C_{k-1} + kC_{k-2} + 1$ [Recursive Relation]

(ii) $C_0 = 1$ and $C_1 = 2$ [initial condⁿ)

$C_4 = ?$ & C_7

⇒ for $k=2$

$c_2 = c_{2-1} + 2c_{2-2} + 1$

$c_2 = 2 + 2c_0 + 1$

$c_2 = 2 + 2 + 1$

$\boxed{c_2 = 5}$

⇒ for $k = 3$

$c_3 = c_{3-1} + 3c_{3-2} + 1$

$c_3 = c_2 + 3c_1 + 1$

$c_3 = 5 + 6 + 1$

$\boxed{c_3 = 12}$

⇒ for $k=4$

$c_4 = c_{4-1} + 4c_{4-2} + 1$

$c_4 = c_3 + 4c_2 + 1$

$c_4 = 12 + 20 + 1$

$\boxed{c_4 = 33}$

For $k=5$

$$C_5 = C_4 + 5C_3 + 1$$

$$C_5 = 33 + 60 + 1$$

$$C_5 = 94$$

For $k=6$

$$C_6 = C_5 + 6C_4 + 1$$

$$C_6 = 94 + 198 + 1$$

$$C_6 = 293$$

for $k=7$

$$C_7 = C_6 + 7C_5 + 1$$

$$C_7 = 293 + 658 + 1$$

$$\boxed{C_7 = 952}$$

Equipotent Sets :→

- Two sets A and B are said to be equipotent or to have same no. of elements or same cardinality, written as $A \simeq B$, if there exist a one-one correspondance b/w them.

MODULE - 2

(#)

⇒ Principle of Inclusion and Exclusion :→

If A and B be any two finite sets, then

$$\boxed{N(A \cup B) = N(A) + N(B) - N(A \cap B)}$$

⇒ for A, B, C

$$\boxed{N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(B \cap C) - N(A \cap C) + N(A \cap B \cap C)}$$

⇒ for n. no.

$$\boxed{N(A_1 \cup A_2 \cup \dots \cup A_r) = N(A_1) + N(A_2) + \dots N(A_r) - \sum_{j,i=1}^{r} ((A_i \cap A_j) + \dots (-1)^{r+1}) (A_1 \cap A_2 \dots A_r)}$$

e.g (i) 1232 Students haven taken in spanish, ~~Further~~
879 = french, 114 - Russian.
103 both in spanish & french.
23 Spanish & Rusian
14 = both french & Rusian

if 2092 taken in atleast one of the french, spanish Russian. How many Student taken a course in both 3 languages.

Soln $n(A \cap B \cap C) = 2092 - 1232 - 879 - 114 + 103 + 23 + 14$

$n(A \cap B \cap C) \Rightarrow 7$

$n(A \cap B \cap C) = n(A \cup B \cup C) - n(A) - n(B) - n(C) + n(A \cap B) + n(A \cap C) + n(B \cap C)$

(iii) Determine set of +ve integer less than and equal 720 which are not divisible by any of 2, 3 and 5.

Sol^n $n(A) = 720/2 = 360$

$n(B) = 720/3 = 240$

$n(C) = 720/5 = 144$

$n(A \cap B) =$ elements divided by 6

$\Rightarrow \frac{720}{6} = 120$

$n(B \cap C) = \frac{720}{15} = 48$

$n(A \cap C) = \frac{720}{10} = 72$

$n(A \cap B \cap C) = \frac{720}{30} = 24$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$\Rightarrow 360 + 240 + 144 - 120 - 48 - 72 + 24$

$n(A \cup B \cup C) = 528$

$\Rightarrow n(A \cup B \cup C)^c = n(U) - n(A \cup B \cup C)$

$\Rightarrow 720 - 528$

$\Rightarrow 192$

Pigeon-Hole Principle

⇒ If n Pigeon holes are occupied by $n+1$ or more pigeons, then atleast one Pigeon hole is occupied by more than pigeon.

⇒ Generalised Pigeon-Hole Principle:→

• If n pigeon holes are occupied by $kn+1$ or more pigeons where k is a +ve integer, then atleast one pigeon hole is occupied by $k+1$ or more pigeons.

e.g. (i) Find min. no. of Students in a class to be ~~[illegible]~~ sure that 3 of them are born in same month.

Soln $n = 12$ (no. of months)

$k+1 = 3$ ~~[illegible]~~ [come in same month].

$kn+1 = ?$

$k+1 = 3$

$k = 2$

$2(12)+1 = 24+1 = 25$

Total min. no. of Students $= 25$

(ii) Find the min. no. of Students needed to guarantee that 5 of them belong to same class.,

Soln $n=4$, Class $=4$

$k+1=5$

$k=4$

$kn+1 = 4(4)+1 = 16+1 = 17$ (min no. of std).

(iii) find the min. no. of students to gurantee that 4 of them were born

(i) on the same day of week.

(ii) on the same month.

Soln (i) $n = 7$, $k+1 = 4$

$k = 3$

$kn+1 = 3(7)+1 = 22$

(ii) $n = 12$, $k+1 = 4$

$k = 3$

$kn+1 = 3(12)+1 = 37$

2nd version of Generalised Pigeon-Hole Principle.

- It states that if ~~the~~ n pigeon are assigned to m pigeon holes, $n > m$, then one of the pigeon holes must contain atleast

$$\left\lfloor \frac{n-1}{m} \right\rfloor + 1.$$

↑ floor fucn.

e.g (i) Show that if 9 colours are used to paint 100 hundred houses, atleast 12 houses will be of the same colour.

Soln $n = 100$, $m = 9$

$$\frac{n-1}{m} = \left\lfloor \frac{\cancel{99}^{11}}{\cancel{9}} \right\rfloor + 1 = 12$$

(ii) How many people among Ten thousand born on one day in the same hour

$n = 10{,}000$, $m = 24$

$\left[\frac{n-1}{m}\right]+1$

$$\left[\frac{10000-1}{24}\right]+1$$

$\Rightarrow [416 \cdot 625]+1$

$\Rightarrow 417$

(iii) How many people atleast in a gp. of 85 people have the same last initial. (alphabets)

Sol^n $n = 85$, $m = 26$

$$\left[\frac{85-1}{26}\right]+1 = \left[\frac{84}{26}\right] = [3 \cdot 2]+1$$
$$= 3+1$$
$$= 4$$

Basic counting Principle :→

I^st counting Principle :→

- If an event can occur in r diff. steps and step 1 can occur in n1 ways, step 2 can occur in n2 ways and so on. Step r can occur in nr ways. Then, the no. of possible events that can occur is $\underbrace{n_1 \times n_2 \times n_3 - - - \cdot n_r}_{\text{Product rule}}$

e.g. A person has to arrange 5 books in a shelf. In how many ways can he do so.

$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

$\Rightarrow$ 120 ways.

(ii) A 5 person committee having members Saniya, Pooja, Ankit, Arijit, Nitish is to select a president, vice-president and Secretary.

(a) How many Selection exclude Saniya.

$\Rightarrow$ $4 \times 3 \times 2 = 24$ ways.

(b) How many Selections are there in which Ankit is President.

P	VP	S
$\uparrow$	$\downarrow$	$\downarrow$
1	4	3

$1 \times 4 \times 3 = 12$ ways

$\Rightarrow$ 2^{nd} counting Principle:-

- Suppose an event $E1$ can occur in n_1 ways, a 2^{nd} event $E2$ can occur in $n2$, & 3^{rd} event $E3$ can occur in $n3$ --- upto event Er can occur in nr ways. if no two events can occur at the same time, then one of the event can occur in $n_1 + n_2 + n_3$ --- nr ways.

e.g (i) Suppose a college has 3 diff. history courses. 4 diff. literature courses and, 2 diff. cyclogy courses. Then the no. of ways a student can choose just 1 of the coursses :-

$\Rightarrow$ $3 + 4 + 2$

$\Rightarrow$ ways for 1 course is $= 9$

(ii) A 5 person committe having members Ankit, Arijit, Pooja, Saniya, Piyush. In how many ways can this occur if either,

(a) $\underbrace{\text{Saniya}}_{(i)}$ or $\underbrace{\text{Piyush}}_{(ii)}$ must be president.

(i) Saniya P.

P → 1

VP → 4

S → 3

Total = 4×3 = 12

(ii) Piyush P

P → 1

VP → 4

S → 3

Total = 4×3 = 12

Total of either Saniya is Pres. or Piyush is President = 12+12 = 24

(b) How many selections are there in which Pooja is S or she is excluded.

Pooja S

P 4

VP → 3

S → 1

Total = 12

Pooja → Excluded

P → 4

VP → 3

S → 2

Total = 24

Total = 12 + 24 = 36

Factorial.

n factorial

- The product of 1^{st} n natural no. is called factorial n, denoted by $n!$ or $\underline{|n}$
- It can also be written as $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

e.g. find the value of Z

$$\frac{1}{4!} + \frac{1}{5!} = \frac{Z}{6!}$$

$$\Rightarrow \frac{1}{24} + \frac{1}{120} = \frac{Z}{720}$$

$$\Rightarrow \frac{120+24}{2880} = \frac{Z}{720}$$

$$\Rightarrow \frac{144}{2880} = \frac{Z}{720}$$

$$\Rightarrow \frac{103680}{2880} = Z$$

$$\Rightarrow Z = 36$$

TUT

Q. How many people among 100 are born in same month.

Soln $n = 100$, $m = 12$

$$\left[\frac{100-1}{m}\right]+1$$

$$\Rightarrow \left[\frac{99}{12}\right]+1$$

$$\Rightarrow [8.2]+1$$

$$= 8+1 = 9$$

Q. How many people among 10,000 born on a day in the same minute.

Soln $n = 10{,}000$, $m = 1440$

$$\Rightarrow \left[\frac{10{,}000-1}{1440}\right]+1$$

$$= [6.9]+1$$

$$= 6+1 = 7$$

Q. Show that if 9 colours are used to pain 100 houses atleast 12 houses will be house of same colour.

Soln $n=100$, $m=9$

$$\Rightarrow \left[\frac{100-1}{9}\right]+1$$

$$\Rightarrow \left[\frac{99}{9}\right]+1$$

$$= 11+1$$

$$\Rightarrow 12$$

Q. Find the min. no. of std. needed to gurantee that 4 of them were born on the same
(i) day of the week.

Soln

$n = 7, \; k+1 = 4$

$k = 3$

$kn+1 = 3 \times 7 + 1$

$\Rightarrow 21+1$

$\Rightarrow 22.$

(ii) In same month

$n = 12, \; k+1 = 4$

$k = 3$

$kn+1 = 12 \times 3 + 1$

$\Rightarrow 36+1$

$\Rightarrow 37$

Q. Find the min. no. of stds. needed to gurantee that 5 of them belong to the same class = 4

$\Rightarrow n = 4, \; k+1 = 5$

$k = 4$

$\Rightarrow kn+1 = 4 \times 4 + 1 = 17$

Q. A 5 person committee having members Saniya, Pooja, Rohini, Ankit, Abijit, Nitish is select, P, VP, S

(a) How many selection include Saniya Pooja.

$P \to 3$

$VP \to 2$

$S \to 3$

$\Rightarrow 3 \times 2 \times 3 = 18$

(5) How many selections in which Arijit is Secretary or excluded.

Arijit (S) | Arijit (Excluded)

P – 4
VP – 3
S – 1
⇒ 12

P 4
VP 3
S → 2
$4 \times 3 \times 2 = 24$

Total = $24 + 12 = 36$

Permutation :→

- A permutation is an arrangement of no. of objects in some definite order taken some or all at a time. The total no. of permutation of n distinct objects taken r at a time is denoted by ^nP_r or $P(n,r)$, where $r \geq 1$ but less than n

$$\boxed{1 \leq r \leq n}$$

$$\Rightarrow \boxed{^nP_r = \frac{n!}{(n-r)!}}$$

⇒ **Note:→**

$$^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! \quad , \quad [0! = 1]$$

e.g. (i) Determine the value of n if $4 \times {}^nP_3 = {}^{n+1}P_3$

Soln $= 4 \times {}^nP_3 = {}^{n+1}P_3$

$$= \frac{4 \times n!}{(n-3)!} = \frac{(n+1)!}{(n+1-3)!}$$

$$= \frac{4 \times n!}{(n-3)!} = \frac{n!(n+1)}{(n-2)!}$$

$$= \frac{4 \times n!}{(n-3)!} = \frac{n!(n+1)}{(n-2)(n-3)!}$$

$$= 4 = \frac{n+1}{n-2}$$

$$\Rightarrow 4n - 8 = n + 1$$

$$3n = 9$$

$$\boxed{n = 3}$$ Ans

(ii) How many variable names of 8 Letters can be formed from the letters, A, B, C, D, E, F, G, H, I. If no letter is repeated.

Soln $n = 9, \quad r = 8$

$${}^nP_r = {}^9P_8 = \frac{9!}{(9-8)!} = \frac{9!}{1!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1}$$

$$= 362,880$$

(iii) How many 6 digits no. can be formed by using the digit 0 – 8, if every no. is to start with 30, with no. digit repeated.

Soln 0, 1, 2, 3, 4, 5, 6, 7, 8.

$3\underbrace{\square\square\square\square}_{^7P_4}$

$$^7P_4 = \frac{7!}{(7-4)!} = \frac{7\times6\times5\times4\times\cancel{3!}}{\cancel{3!}}$$

$\Rightarrow 840$

(iv) In how many ways 5 diff. microprocessor books, and 4 diff. D.E books be arranged in a self so that all the four DE books ~~be arranged~~ are together.

Soln

5 → MP. books

$\boxed{4 \rightarrow \text{D.E books}}$

5 + 1 unit

$\Rightarrow 6! \times 4!$

$\Rightarrow 17,280$

(V) How many permutations can be made out of the letter of the word COMPUTER. How many of these

(i) begin with C

$\boxed{C}$ _ _ _ _ _ _ _

$\Rightarrow 7! = 5040$

(ii) end with R

$7! = 5040$

(iii) begin with c and end with r.

$\Rightarrow 6! = 720$

(iv) Permutation in which c and r are occupied end places

$\Rightarrow 6! \times 2!$

$\Rightarrow 1440$

Note :→

Permutation when all the objects are not distinct.

$$\frac{n!}{n_1! \, n_2! \cdots n_r!}$$

• The no. of permutation of n object of which n_1 object are of one kind, n_2 are of 2nd kind, n_3 objects are of 3rd kind and so on ---. upto n_t objects of t kinds

Then no. of permutation is given by

$$\boxed{\frac{n!}{n_1! \, n_2! \cdots n_t!}}$$

where $n_1 + n_2 + n_3 \cdots n_t = n$

e.g.(i) Determine the no. of permutation that can be made out of the letters of word PROGRAMMING

$$\Rightarrow \frac{n!}{n_1!\, n_2 \cdots n_r!}$$

$$\Rightarrow \frac{11!}{2!\,2!\,2!} \Rightarrow 4989,600.$$

(ii) There are 4 blue, 3 Red, 2 black pen in a box. These are drawn one by one. Determine all the diff. permutations.

Soln Total = 9.

$$\Rightarrow \frac{9!}{4!\,3!\,2!} = 1260$$

Note

Permutation with Repeated objects:

- The no. of different permutation of n distinct objects taken r at a time, when every object is allowed to repeat any no. of times is given by - n^r

e.g. (i) How many 4 digit no. can be formed by using 2, 4, 6, 8 when repeation is allowed.

$\Rightarrow$ $4^4 = 256.$

Circular Permutation:→

- The no. of circular permutation of n objects taken all n at a time is $(n-1)!$

e.g. (i) How many ways can 5 children arrange themselves in a ring.

Soln $n=5$

$$(n-1)! = (5-1)! = 4! = 24.$$

(ii) In how many ways 10 programmers can sit on a round table to discuss the project so that project leader and a particular programmer always sit together.

⇒ 10 Programmer

2 → P.L & P.P.

$$\Rightarrow (10-2+1) = 9$$

$$(n-1)! = 8!$$

$$\Rightarrow 8! \times 2! = 80,640$$

Combination:→

- A Combination is a selection of sum or all object from a set of given objects where order of the object doesn't matter, then the no. of combinations of n object taken r at a time is representing by

- nCr or $C(n,r)$ or $\binom{n}{r}$

$$\Rightarrow \boxed{nCr = \frac{n!}{r!(n-r)!}}$$

Note

(i) $^nC_n = \dfrac{n!}{n!\,(n-n)!} = \dfrac{n!}{n!\,0!} = 1$

(ii) $^nC_0 = \dfrac{n!}{0!\,(n-0)!} = \dfrac{n!}{1 \times n!} = 1$

e.g (i) Determine the value of n, $^nC_{n-2} = 10$

Sol^n $^nC_{n-2} = 10$

$\Rightarrow \dfrac{n!}{(n-2)!\,(n-(n-2))!} = 10$

$\Rightarrow \dfrac{n!}{(n-2)!\,2!} = 10$

$\Rightarrow \dfrac{n\,(n-1)\,\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$

$\Rightarrow n^2 - n = 20$

$\Rightarrow n^2 - n - 20 = 0$

$\Rightarrow n^2 - 5n + 4n - 20 = 0$

$\Rightarrow n(n-5) + 4(n-5) = 0$

$\Rightarrow (n-5)(n+4) = 0$

$\Rightarrow n - 5 = 0$ | $n + 4 = 0$

$\boxed{n = 5}$ | $\boxed{n = -4}$ (Rejected)

(ii) How many ways can be select a software development gp. of one project leader. 5 project programmers, 6 data entry operators from a gp. of 5 project leader, 20 programmers, 25 data entry operators.

Soln ~~Total~~ 5 P.L → 1 P.L

20 Prog → 5 Prog

25 data entry → 6 data entry.

$$\Rightarrow 5C_1 \times 20C_5 \times 25C_6$$

$$\Rightarrow \frac{5!}{1!(5-1)!} \times \frac{20!}{5!(20-5)!} \times \frac{25!}{6!(25-6)!}$$

$$\Rightarrow 5!$$

(iii) From 10 programmers, In how many ways can 5 be selected when (i) a particular is included everytime.

$$\Rightarrow 9C_4 = \frac{9!}{4!(9-4)!} = \frac{9!}{4!\,5!} = 126$$

(ii) a particular Pro. is not included at all.

$$\Rightarrow 9C_5 = \frac{9!}{5!(9-5)!} = \frac{9!}{5!\,4!} = 126$$

(iv) consider the six digits no - 2, 3, 4, 5, 6, 8 and repeation of digits are allowed.

(i) How many 3 digit no. can be formed.

$\Rightarrow$ $\boxed{\ }$ $\boxed{\ }$ $\boxed{\ }$

6 6 6

$6^3 = 216$

(ii) How many 3 digit no. must contain the digit 5.

$\Rightarrow$ $\boxed{5}$ $\boxed{\ }$ $\boxed{\ }$

Total - Excluding 5

$216 - 125 =$

(V) In how many ways, a committee of 5 members can be selected from 6 men & 5 women consisting of 3 men and 2 women.

$\Rightarrow$ $6C_3 \times 5C_2$

$$\frac{6!}{3!\,3!} \times \frac{5!}{2!\,3!}$$

(VI) The Question Paper of maths. contains 2 ~~do~~ Ques. divided into 2 gp. of 5 Q. each. In how many ways can an examinee ans. 6 Ques taking atleast 2 Ques. from each gp.

Group 1 — 5

Gp. 2 — 5

Case 1

G.P1 $\rightarrow$ 2 $\rightarrow$ $5C_2$

G.P2 $\rightarrow$ 4 $\rightarrow$ $5C_4$

$\Rightarrow$ $5C_2 \times 5C_4$

Case 2 G.P 1 → 3 → $5C_3$

G.P 2 → 3 → $5C_3$

$5C_3 \times 5C_3$

Case 3 G.P 1 → 4 → $5C_4$

G.P 2 → 2 → $5C_2$

$5C_4 \times 5C_2$

(VII) Out of 5 men and 2 women, a comittee of 3 is to formed. In how many ways, can it be formed, if atleast one woman is to included.

⇒ 5 men and 2 women

comittee = 3

Case 1 2M 1W

$5C_2 \times 2C_1$

Case 2 1M 2W

$5C_1 \times 2C_2$

(VIII) Out of 9 girls and 15 boys. How many diff. comittee can be formed Each consist of 6 boys and 4 girls.

⇒ 9 girls and 15 boys

C → 6 boys and 4 girls

1st $\quad {}^{15}C_6 \times {}^{9}C_4$

now, boys = 9 & girls = 5

2nd $\quad {}^{9}C_6 \times {}^{5}C_4$

Combination with Repeatition formula

⇒ To find out the no. of combination when repeatition is allowed is

$$\boxed{C(n+r-1, r) = {}^{n+r-1}C_r}$$

where n = Total no. of objects in a set.

r = no. of objects that can be selected from a set.

e.g. The no. of ways to choose 3 out 7 days when repeatition is allowed.

⇒ $n = 7$, $r = 3$

$${}^{7+3-1}C_3$$

$$\Rightarrow {}^{9}C_3 \Rightarrow \frac{362880}{4320}$$

$$\Rightarrow 84$$

(VIII) Out of 9 girls and 15 boys. How many diff. comittee can be formed Each consist of 6 boys and 4 girls.

⇒ 9 girls and 15 boys

$C \to$ 6 boys and 4 girls

1st $\quad {}^{15}C_6 \times {}^{9}C_4$

Now, boys = 9 & girls = 5

2nd $\quad {}^{9}C_6 \times {}^{5}C_4$

Combination with Repeatition formula

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e.g. The no. of ways to choose 3 out 7 days when repeatition is allowed.

⇒ $n = 7$, $r = 3$

$${}^{7+3-1}C_3$$

$$\Rightarrow {}^{9}C_3 \Rightarrow \frac{362880}{4320}$$

$$\Rightarrow 84$$

unit-1

Theoram

Schroeder - Bernstein Theoram :→

Statement → If A and B are two sets
cardinality
such than $\sigma(A) \le \sigma(B)$ ff A and
$\sigma(B) \le \sigma(A)$, then $\sigma(A) = \sigma(B)$

OR

If A and B are two sets. Each of which is numerically equivalent to the subset of other than A is numerically equivalent to set B

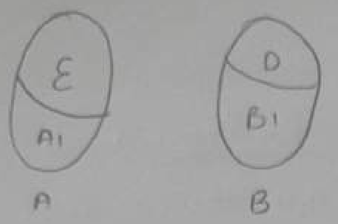
Proof Let A and B are two sets such that $n(A) \le n(B)$ and $n(B) \le n(A)$ then, we have to show $n(A) = n(B)$.

⇒ Since $car(A) \le car(B)$, therefore by defn of set, A is numerically equivalent to subset of B i.e 'D'.

→ There exist
∴ ∃ a mapping $f: A \to D$, which is one-one and onto.
Also, we have $f(A) = D$.

Similarly, if $card(B) \le card(A)$, therefore by defn of set, B is numerically equivalent to set of A.

∴ ∃ a mapping $g: B \to E$, which is one-one and onto, also, we have $g(B) = E$.



Let $A - \mathcal{E} = A_1$
$f(A_1) = B_1$

$g(B_1) = A_2$

$f(A_2) = B_2$

Moreover, let

$A - \bigcup_{n=1}^{\infty} A_n = P$ and $B - \bigcup_{n=1}^{\infty} B_n = M$

Since $f(A_n) = B_n$ and f is 1-1

$\therefore A_n \sim B_n$ for $n = 1, 2, 3 - - -$

Since $B_1 \subset B \Rightarrow g(B_1) \subset g(B)$

$\Rightarrow A_2 \subseteq E$

But $A_1 = A - \mathcal{E}$ then $A_1 \cap A_2 = \phi$

i.e A_1 and A_2 are disjoint Sets.

Again, $B_1 \cap B_2 = f(A_1) \cap f(A_2)$

$= f(A_1 \cap A_2)$

$\Rightarrow f(\phi) = \phi$

i.e B_1 & B_2 are disjoint Sets.

Next, $A_2 \cap A_3$

$= g(B_1) \cap g(B_2)$

$= g(B_1 \cap B_2)$

$\Rightarrow g(\phi) = \phi$

$\Rightarrow A_1, A_2, A_3 - - \cdot A_n$ and

$B_1, B_2 - - \cdot B_n$ are disjoint sets.

Since $A_n \cup B_n$ for each n $\cup A_n \sim \cup B_n$

Let $g: M \longrightarrow P$ be any mapping

given by $g(m) = p \ \forall \ m \in M, \ p \in P$.

clearly, mapping g is 1-1 and onto.

Now, each A is decompose into two disjoint set.

$$A = P \cup \left(\bigcup_{n=1}^{\infty} A_n \right)$$

and $B = M \cup \left(\bigcup_{n=1}^{\infty} B_n \right)$

$P \sim M$ and $\bigcup_{n=1}^{\infty} A_n \sim \bigcup_{n=1}^{\infty} B_n$

$\therefore A \sim B$

UNIT-3

Introduction:→

Mathematical logic or logic is the study of reasoning the main objective of logic is to formulate rules which may be helpful in taking decisions whether any argument or reasining is valid.

Prepositions | Statement:→

- A preposition or statement is a declarat-ive statement which is true or False But not both.

⇒ For instance, the foll. are prepositions :-

(i) Delhi is the capital is India [True]

(ii) $1+1 = 2$ [True]

(iii) $2+1 = 4$ [False]

⇒ However the foll. are not prepositions :-

(i) what is your name?

(ii) Do your homework.

(iii) x is an even no.

Note →
- The truth or Falsehood of a preposition is called its Truth value.
- The lower case letters starting from p onwards are used to represent

Prepositions.

e.g. (i) p: $2+2=4$

(ii) q: Delhi is in Asia.

- The truth value of a preposition is True denoted by T, if it is a preposition and False denoted by F, if it is a False preposition.

Prepositional Logic:→

- The area of logic that deals with preposition is called prepositional logic or Prepositional calcus.

compound Preposition

- Two or more prepositions when combined by various connectivities into a single composite sentence such composite preposition are called compound prepositions.

e.g. • A triangle is equilateral if and only if its 3 sides are equal.

- John is smart or he studies every night.

- Earth is round and revolves around the sun.

(#) Logical connectives :→

- The Particular words and Symbols used to join two or more preposition into a single form or compound preposition are called logical connectives.

(#) Logical connectives	Symbol	uses.
1. conjunction / And / Join	$\wedge$	$p \wedge q$
2. Disjunction / or / meet	$\vee$	$p \vee q$
3. Negation	$\neg$ or $\sim$	$\sim p$
4. Conditional (if -- then --)	$\rightarrow$	$p \rightarrow q$
5. Biconditional (if and only if)	$\longleftrightarrow$	$p \longleftrightarrow q$
6. NAND (Not + And)	$\uparrow$	$p \uparrow q$
7. NOR (NOT + OR)	$\downarrow$	$p \downarrow q$
8. XOR	$\oplus$	$p \oplus q$

Fundamendal connectors:-

(i) conjunction:-

- Any Two Preposition can be combined by the word and to form a compound preposition called the conjuction of the original preposition.

- Assume P and Q be two prepositions conjunction of P & Q to be a preposition which is true when both P and Q are true, other-wise False.

- It is denoted by $p \wedge q$.

Truth Table :-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(ii) Disjunction :→

- Any two preposition can be combined by the word or to form a compound preposition called the disjunction of the original preposition.

- Assume P and Q be two prepositions Disjunction of P and Q to be a preposition which is true when ~~are~~ either one or both P and Q are True and it is false when P and Q are false.

- It is denoted by $p \vee q$

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(iii) Negation :→

- It means opposite of original Statement

- Assume P be a preposition then negation of P to be a preposition which is True when P is False and it is false when P is True.

- It is denoted by $\sim p$.

Truth Table

p	$\sim p$
T	F
F	T

e.g. Generate the T.T for $\sim(p \wedge \sim q)$

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

(iv) NAND

- It means negation after AND of two statements.
- Assume P and Q be two prepositions NAND of P & Q to be a preposition which is false when both P and Q are True Otherwise it is True.
- It is denoted by $p \uparrow q$.

Truth Table.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

(V) NOR:-

- It means negation after OR of two Statements.
- Assume P and Q be two Prepositions then NOR of P and Q to be a preposition which is true when P and Q are False otherwise it is False.

- It is denoted by $P \downarrow q$

Truth Table

P	q	$P \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

(VI) XOR:-

- Assume P and q be two prepositions then XOR of P and q is true if P is true or q is True but not both and vice-versa.

- It is denoted by. $P \oplus q$

Truth Table:

P	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

e.g. Generate the Truth Table of
$p \oplus q \oplus r$

p	q	r	$p \oplus q$	$p \oplus q \oplus r$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	T	T
F	T	T	T	F
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Some other connectors :-

(i) conditional

- Statements of the form if P then q are called conditional statements
- It is denoted by $P \rightarrow q$ and read as P implies q or Q is necessary for P or P is sufficient for q.
- Conditional St. is true if both P and Q are True or if P is False. It is False if P is True and Q is False.
- The Prepositions P is called hypothesis and preposition q is called conclusion.

Truth Table:-

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(✱) variations in conditional Statements:-

(i) contra positive:-

- The preposition negation of q implies negation of p is called contrapositive of p implies q.

(ii) converse:-

- The preposition $q \to p$ is called converse of $p \to q$.

(iii) Inverse:-

The prepos^n $\sim p \to \sim q$ is called inverse of $p \to q$.

⇒ Equivalent:- $p \to q$ and its contrative $\sim q \to \sim p$ are equivalent.

Show that

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Show that $q \to p$ and $\sim p \to \sim q$ is not equivalent to $p \to q$.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$	$q \to p$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Bi conditional :→

- Biconditional statements of the form if and only if are Bi-conditional statement
- It is denoted as $p \leftrightarrow q$ and read as p if and only if q.

Truth Table :-

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- The preposition p if and only if q is True if p & q have same Truth values. and is false if p & q do not have same Truth values.

e.g. Prove that p if & only if q is equivalent to $(p \to q) \wedge (q \to p)$

P	q	$P \to q$	$q \to P$	$(P \leftrightarrow q)$	$(P \to q) \wedge (q \to P)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Tautologies :-

- A preposition P is a Tautology if it is True under all circumstances. It means it contains only T in the final column of its Truth Table.

e.g. Prove that the statement $(P \to q) \leftrightarrow (\sim q \to \sim P)$ is a Tautology.

P	q	$\sim q$	$\sim P$	$P \to q$	$\sim q \to \sim P$	$(P \to q) \leftrightarrow (\sim q \to \sim P)$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

contradiction :-

A preposition P is a contradiction if it is False under all circumstances. It means it contains only F in the final column of its Truth Table.

e.g. Show that $p \wedge \sim p$ is contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Contingency :→

- A statement that can be either True or False depending upon the Truth values of its variables is called contigency.

e.g. (i) Prove that the $(p \to q) \to (p \wedge q)$ is contigency.

p	q	$p \to q$	$p \wedge q$	$(p \to q) \to (p \wedge q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

e.g. Determine whether is Tautology, contradiction and contigency.

$p \to (p \vee q)$

(i) $p \to (q \to p)$; $p \to (p \vee q)$

(ii) $\sim (p \wedge q) \vee (\sim p \vee \sim q)$

(iii) $p \to (q \to r) \to ((p \to q) \to (p \to r))$.

(i)

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

$\Rightarrow$ Tautology.

(ii)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$	$\sim(p \wedge q) \vee (\sim p \vee \sim q)$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

$\Rightarrow$ contigency.

(iii)

p	q	r	$q \to r$	$p \to q$	$p \to r$	$p \to (q \to r)$	$(p \to q) \to (p \to r)$	$p \to (q \to r) \to (p \to q) \to (p \to r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

$\Rightarrow$ Tautology

e.g. Generate the truth Table for the following :→

(i) $A \oplus B \oplus C$

A	B	C	$A \oplus B$	$A \oplus B \oplus C$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Logical Equivalence:→

Two prepositions $P(p, q, r \ldots)$ and $Q(p, q, r \ldots)$ are said to be logically equivalent or simply equivalent or equal denoted by $P(p,q,r) \equiv Q(p,q,r \ldots)$ if they have identical truth Table.

Law of Logical Equivalence:→

(i) Idempotent Laws:→

- $p \vee p \equiv p$
- $p \wedge p \equiv p$

(ii) Associative Laws:→

- $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(iii) Commutative Laws:→

- $p \vee q \equiv q \vee p$
- $p \wedge q \equiv q \wedge p$

(iv) Distributive Laws:→

- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(V) Identity laws:→

- $p \vee F \equiv p$
- $p \wedge T \equiv p$
- $p \vee T \equiv T$
- $p \wedge F \equiv F$

(VI) Involution Law:→

$\sim(\sim p) \equiv p$

(VII) Complement Law:→

- $p \vee \sim p \equiv T$
- $p \wedge \sim p \equiv F$
- $\sim T \equiv F$
- $\sim F \equiv T$

(VIII) De-Morgan's Law:→

- $\sim(p \vee q) \equiv \sim p \wedge \sim q$
- $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Q. Write an equivalent formula for

$p \wedge (r \leftrightarrow q) \vee (s \leftrightarrow p)$ does not involve biconditional.

$\Rightarrow \quad (p \leftrightarrow q) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Q. generate T.T for $p \rightarrow q$ and $\sim p \vee q$

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Q. $p \wedge (r \leftrightarrow s) \vee (s \leftrightarrow p)$ does not involve biconditional and conditional.

$\Rightarrow \quad p \wedge (r \leftrightarrow s) \vee (s \leftrightarrow p)$

$= \quad p \wedge (r \rightarrow s) \wedge (s \rightarrow r) \vee (s \rightarrow p) \wedge (p \rightarrow s))$

$= p \wedge (\sim r \vee s) \wedge (\sim s \vee r)) \vee ((\sim s \vee p) \wedge (\sim p \vee s))$

Q. Translate the foll. statements in preposition logic

(i) if catalog is correct then if the seeds are planted in April, flowers will bloom in july.

Soln p = catalogue is correct

q = seeds are planted in April

r = flowers will bloom in july.

$\Rightarrow p \rightarrow (q \rightarrow r)$

(ii) If John is elected CR then either Mary is elected Treasurer or Alice is elected vice Treasurer.

p = John is elected

q = Mary is elected Treasurer

r = Alice is ~~Alice~~ elected vice Treasurer

$$p \longrightarrow (q \vee r)$$

(iii) Either taxes are increased or if expenditure rises then ~~depo~~ debt is raised.

p = taxes are increased

q = expenditure rises

r = debt is raised

$$\cancel{(p \vee q) \rightarrow r} \quad p \vee (q \rightarrow r)$$

(#) Argument :-

- An argument is an assertion that a given set of prepositions $P_1, P_2, P_3 -- P_n$ called premises yields another preposition Q called conclusion. Such an argument is denoted by $P_1, P_2, P_3 -- P_n \vdash Q$.

(#) Premises :-

- The prepositions which are assumed for accepting the conclusion are called premises of that argument.

Conclusion :→

- It is the preposition that is asserted on the basis of other preposition of the argument.

Valid Argument :→

- An argument is called valid argument if the conclusion is True whenever all the premises are True.

- The argument is also valid if and if are called the AND of the gp of prepositions implies conclusion is a tautology i.e $P(P_1, P_2 -- P_n) \rightarrow Q$ is a tautology where $P(P_1, P_2, -- P_n)$ is the gp. of preposition and Q is conclusion.

Falacy Argument :→

- An argument is called Falacy if it is not valid.

e.g. Show that the rule of hypothetical Syllogism is valid.

$$p \rightarrow q$$
$$q \rightarrow r$$
$$\therefore p \rightarrow r$$

Soln

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
F	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	T
F	F	T	T	T	T
F	F	T	T	T	T

$\Rightarrow$ $p \to r$ is true in lines 1, 2, 4, 6, 7, 8

$\Rightarrow$ $q \to r$ is true inlines 1, 3, 4, 5, 7, 8

$\Rightarrow$ Both are true in 1, 4, 7, 8

$\Rightarrow$ Given argument is valid.

$(p \to q) \wedge (q \to r) \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

e.g. Consider the foll. argument and determine whether it is valid or not.

Either I will get good marks or I will not graduate. If I did not graduate I will go to Canada. I get good marks. Thus, I would not go to Canada.

Soln p : I will get good marks.

q : graduated.

r : will go to canada.

$\Rightarrow$ $p \vee \sim q$

$\sim q \rightarrow r$

p

$\therefore \sim r$

from Tautology

p	q	$\sim q$	r	$\sim r$	$p \vee \sim q$	$\sim q \rightarrow r$	$[(p \vee \sim q) \wedge (\sim q \rightarrow r) \wedge p] \rightarrow \sim r$
T	T	F	T	F	T	T	
T	T	F	F	T	T	T	
T	F	T	T	F	T	T	
F	T	F	T	F	F	T	
T	F	T	F	T	T	F	
F	T	F	F	T	F	T	
F	F	T	T	F	T	T	
F	F	T	F	T	T	F	

$p \vee \sim q$ is true in lines 1, 2, 3, 5, 7, 8

$\sim q \rightarrow r$ is true in lines 1, 2, 3, 4, 6, 7

p is true in 1, 2, 3, 5

All three are True in 1, 2, 3
$\sim r$ is true &

$\therefore$ Not valid.

Satisfiablity:→

- A compound preposition is satisfiable if there is an assignment of truth values to its variables that makes it true when no such assignments exist. i.e when the compound preposition is false for all assignment of Truth to its valiables, the compound preposition is unsatisfiable.

Note

- The other way of satisfiablity interprepation is a prepositional statement is satisfiable if and only if its truth table is not contradiction.

eg. Test the validity of the foll. argument.
if two sides of a Δ are equal then the opp. angle are equal. Since two sides of Δ are not equal. Therefore opp. angles are not equal.

p = two sides of Δ are equal
q = opp angle are equal.

$p \to q$

$\sim p$

$\therefore \sim q$

p	q	~p	~q	p→q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

p→q is true in 1, 3, 4
~p true in 3, 4
Both true in 3, 4
~q is true in 4
not valid

Quantifiers :-

Each Statement that contains a word indicating Quantity such as all, every, some, many, more, few etc. Such words are Quantifiers.

There are two types of Quantifiers :-

(i) Universal :-

If $P(x)$ is a preposition over the universe U then it is denoted as $\forall x, P(x)$ and read as - for every $x \in U$. $P(x)$ is true. The Quantifier $\forall$ is called universal Quantifier.

(ii) Existential :-

If $P(x)$ is a preposition over the universe U then it is denoted as $\exists x, P(x)$ and read as, there exist atleast one value in the universe of variable x such that $P(x)$ is true. The Quantifier $\exists$ is called Existential Quantifier.

Note

$$\sim [\forall x, P(x)] \equiv \exists x, \sim P(x)$$

$$\sim [\exists x, P(x)] \equiv \forall x, \sim P(x)$$

UNIT-4

GROUP THEORY

Binary Operation:-

- Consider a non-empty set and a fuc^n f such that $f: A \times A \to A$ is called a binary operation on A. and it may be written as $a * b$

e.g. Consider the set N of the integers
(i) Add^n & Multiplication are operation on N
$(N, +)$, $(N, \cdot)$
(ii) Sub & Division are not operation on N.
$(N, -)$, $(N, \div)$

Algebric Structure:-

- The Algebric Structure is a type of non-empty A which is equipt with one or more than one Binary operation.
- It is denoted as $(A, *_1, *_2, *_3, \ldots *_n)$ and called Algebric Structure.

Table of Operation:-

consider a non-empty finite set $A = A_1, A_2 \ldots A_n$. A binary Operation * A can be describe by means of a Table

$*$	A_1	A_2	---	A_n
A_1	$A_1 * A_1$	$A_1 * A_2$	---	$A_1 * A_n$
A_2	$A_2 * A_1$			
⋮				
A_n				

Close e.g.

Consider a set A which is equal = -1, 0, 1
Determine whether A is closed under the operation (i) Add^n (ii) Multiplication.

$\Rightarrow$ $A = \{-1, 0, 1\}$

(i)

+	-1	0	1
-1	-2	-1	0
0	-1	0	1
1	0	1	2

(ii)

×	-1	0	1
-1	1	0	-1
0	0	0	0
1	-1	0	1

Properties of Binary operation :-

(i) closure- $A * b \in A \ \forall a, b \in A$

(ii) Associative - $a, b, c \in A$, $(a*b)*c = a*(b*c)$

(iii) Identity :-
$a * e = a = e \times a \ \forall a \in A, e \in A$

(iv) Inverse :-
$a * b = e = b * a \ \forall a, b \in A$

(V) commutative
$a * b = b * a \ \forall a, b \in A$.

Tut

Q. Test the validity of foll. argument. If I will be selected in IAS examination. then I will not be able to go to london. Since I am going to london, I will not be selected in IAS examination.

$\Rightarrow$ P = Selected in IAS exam

Q = go to london.

$$p \to \sim q$$
$$q$$
$$\therefore \sim p$$

p	q	$\sim p$	$\sim q$	$p \to \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$p \to \sim q$ true in 2, 3, 4

q true in 1, 3

Both are true, 3

$\sim p$ is true in 3

$\Rightarrow$ valid

Q. Test the validity of foll. argument.
If it rains, then it will be cold. If it is cold then I shall stay at home, since it rains. therefore, I shall stay at home.

P = It is rain

Q = It will be cold

r = Stay at home

$$p \to q$$
$$q \to r$$
$$p$$
$$\therefore r$$

p	q	r	$p \to q$	$q \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	T

$p \to q$ true in 1, 2, 5, 6, 7, 8

$q \to r$ true in 1, 3, 4, 5, 7, 8

p is true in 1, 2, 3, 4

Both are true in 1, 3

r is 1, 3

$\Rightarrow$ valid.

Q. If 6 is even then 2 does not divide 7. either 5 is not prime or 2 divides 7 But 5 is prime. Therefore 6 is odd.

p = 6 is even

q = 2 divides 7

r = 5 is Prime

$p \to \sim q$

~~$q \vee r$~~ $\sim r \vee q$

r

$\sim p$

p	q	r	$\sim q$	$\sim p$	$p \to \sim q$	$\sim r \vee q$
T	T	T	F	F	F	T
T	T	F	F	F	F	T
T	F	T	T	F	T	F
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	F
F	F	F	T	T	T	T

$p \to \sim q$ is true in 3, 4, 5, 6, 7, 8

$\sim r \vee q$ ~~after~~ is true in 1, 2, 4, 5, 6, 8, 8

r is true in 1, 3, 4, 7

$\therefore$ all are true in = 3, 4

$\sim p$ is true in 5

$\Rightarrow$ not valid

e.g. Consider the b.o $*$ on the set of integers defined $a*b = \frac{ab}{4}$. Determine the identity for the B.O if its exit.

Soln $a*b = \frac{ab}{4}$

⇒ identity

let $e \in Z_+$ and $a \in Z_+$

$a*e = a = e*a \ \forall a, e \in Z_+$

$a*e = a$ | $e*a = a$

$\frac{ae}{4} = a$ | $\frac{ea}{4} = a$

$\boxed{e=4}$ | $\boxed{e=4}$

⇒ inverse $a*b = e$

$\frac{ab}{4} = 4$

$b = \frac{16}{a} \ \forall a, b \in Z_+$

⇒ Inverse doesn't exist.

e.g consider the bo start on Q. ~~Fraction~~ Rationals

defined by $a*b = a+b-ab \ \forall a, b \in Q$

determine whether $*$ is associative

⇒ $a*b = a+b-ab \ \forall a, b \in Q$

$(a*b)*c = (a+b-ab)*c$

⇒ $a+b-ab+c-(a+b-ab)*c$

$= a+b+c-ab-bc-ac+abc$

Sol $a*(b*c) = a*(b+c-bc)$

$$= a+b+c-bc-a(b+c-bc)$$
$$= a+b+c-ab-ac-bc+abc$$

#	Quasi	Semi	Monoid	Group	Abelian group
closure	✓	✓	✓	✓	✓
Associative	×	✓	✓	✓	✓
Identity	×	×	✓	✓	✓
Inverse	×	×	×	✓	✓
commutative	×	×	×	×	✓

e.g. $a*b = \frac{ab}{4}$ $\forall a, b \in R - \{0\}$ for Abelian gp.

(i) closure

$a*b = \frac{ab}{4} \in \mathbb{R}$

(ii) Associative

$(a*b)*c = a*(b*c)$

LHS $(a*b)*c = \left(\frac{ab}{4}\right)*c = \frac{abc}{4}$

$a*(b*c) = a*\left(\frac{bc}{4}\right) = \frac{abc}{4}$

(iii) Identity:-

$a*e = a = e*a$

$a*e = a$

$\frac{ae}{4} = a \Rightarrow \boxed{e=4}$

Inverse

$a * b = e = b * a$

$\frac{ab}{4} = 4 \Rightarrow b = \frac{16}{a}$

Commutative :-

$a * b = b * a$

$a * b = \frac{ab}{4}$

$b * a = \frac{ba}{4}$

e.g consider an algebric st. G^* where $G = \mathbb{R}^2$ and $*$ is b.o defined by $(a, b) * (c, d) = (a + bc, bd)$

Show that G^* is not an abelian gp.

Finite GP → A gp $(G, *)$ is called finite gp, if G is a finite Set.

Infinite GP → A gp $(G, *)$ is called infinite gp, if G is a infinite Set.

Order of a group :→

- The order of a gp. G is the no. of elements in the gp G.
- It is denoted as $|G|$.

Subgroup :-

- Let G be a gp with respect to the Binary op. $*$ and S be a Subset of G then $(S, *)$ is called a Subgroup, if it satisfies foll. condⁿ :-

(i) ~~...~~ The op. $*$ is closed op. on S

(ii) The op. $*$ is an associative operation.

(iii) As e is an identity element belonging to G then it must belong to set S i.e the identity element of $(G, *)$ must belong to $(S, *)$

(iv) for every element a belong to s, a^{-1} also belong to s

e.g. let $(G, +)$ be gp. where G is set of all integers then $(2Z, +)$ is a subgroup of gp. G.

Soln

e.g. let $(Z, +)$ be a gp. where Z = set of integers and + is add op. determine whether the foll subset of G. is a subgp. of G or not

Soln G_1 = set of non-negative integers.

$$(G_1, +) \leq (Z, +)$$

(i) closure:-

let $a, b \in G_1$

S.T $a+b \in G_1 \ \forall \ a, b \in G_1$

$\therefore$ + is closed operation

(ii) Associativity:-

(iii) Identity:-

The element 0 is the identity element. Hence $0 \in G_1$

(iv) Inverse.

$$a * b = e$$

$$a + b = 0$$

$$b = -a$$

$\Rightarrow$ The inverse of every element $a \in G_1$ is $-a$, belonging to G_1. Hence inverse of every element does not exist as $-a \notin G$.

Cyclic Group :→

- Let G be a gp. then G is said to be a cyclic gp. If there exist g belonging to G such that every element in G is of the form g^n for some integer n. The element g is called the generator of the G or that the gp is generated by g.

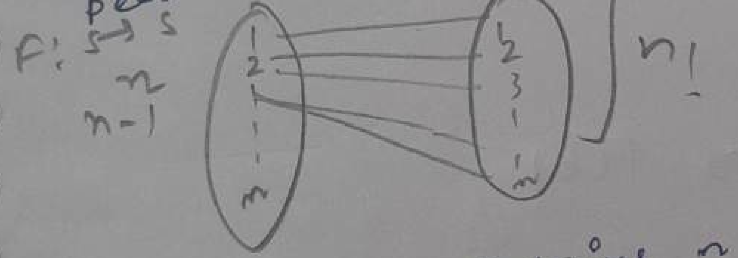
e.g. The integer under add^n is a cyclic gp. the no. 1 is a generator. This is becz any n in the integer we have $n = n \times 1$ also -1 is a generator.

$(Z,+)$ — $1 = 1$; $(-1)^2 = 1$

a^n
na
$n \cdot 1$

Permutation G.P :→

- Let S be an non-empty then a one-one, onto fuc^n f from S onto itself is called a permutation on X.



- If a set contains n elements. There are $n!$ different permutations on S.

- Let S is equal to $a_1, a_2 - - a_n$ be a finite set having n distinct elements and $f: S \to S$ be a one and onto mapping, then f is a permutation of degree n
Let $f(a_1) = b_1$, $f(a_2) = b_2$ and so on. $f(a_n) = b_n$

where $b_1, b_2 --- b_n$ is some arrangement of the elements of the set S.

Coset:→ $(G, *)$, $H \leq G$

left coset → $a * H = \{ah : a \in G, h \in H\}$

Right coset → $H * a = \{ha : a \in G, h \in H\}$

Normal subgroup:→

- A subgroup H of G is a normal subgp. if $a^{-1}Ha$ is contained in H for every $a \in G$ or equivalently if $aH = Ha$ ie; if the right and left coset coincide

$$\boxed{\begin{array}{l} aH = Ha \\ a^{-1}Ha \subseteq H \end{array}}$$

Morphism → any fuc^n.

Homomorphism:→

$(G_1, *_1)$ and $(G_2, *_2)$

$$f : (G_1 \longrightarrow G_2)$$

$$f(a *_1 b) = f(a) *_2 f(b)$$

e.g. (i) $f : R \rightarrow R$

$$f(x) = kx$$

$$f(x+y) = k(x+y)$$

$$\Rightarrow kx + ky$$

$$= f(x) + f(y)$$

Hence it is Homomorphism.

Isomorphism:→

- $f : (G_1, *_1) \longrightarrow (G_2, *_2)$
- one-one
- onto

$$G_1 \cong G_2$$

Automorphism:→

$f : G \longrightarrow G$

Ring Theory:→

$(R, +, \cdot)$

$(R, +)$ is .

Type:→

(i) Commutative Ring:→

- A Ring ~~(R,+,·)~~ $(R,+,\cdot)$ is called a commutative ring if it holds the commutation law under the op. $\underset{\text{multiplication}}{\times}$ i.e $a\cdot b = b\cdot a \;\forall\; a,b \in R$.

(ii) Ring with Unity:→

- A Ring $(R,+,\cdot)$ is called a ring with unity, if it has multiplicative identity i.e, $a\cdot e = e\cdot a = a \;\forall\; a\in R$.

(iii) Ring without Zero divisor:→

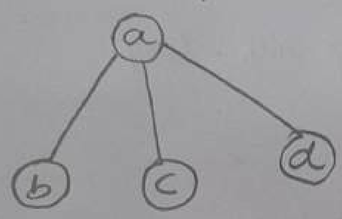
UNIT-5

Graph:→

A graph G has pair G(V, E), V is a non-empty finite set whose elements are called vertices or nodes or point and E is a another set whose elements are called Edges.

The Graph G with vertices V and Edges E is written as G = (V, E) or G(V, E)

e.g Draw a Graph G where V = {a, b, c, d}
E = ((a, b), (a, c), (a, d)}



Basic Terminology:→

⊙ Trivial Graph:→
- One vertex
- no Edge.

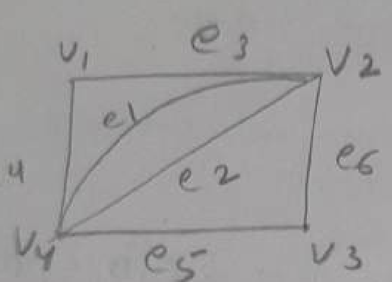
⊙ Null Graph:→
∅ Vertex but no edge. v_1 · · v_2 · v_3 · v_4.

⊙ Empty Graph:-
- 0 Vertex and 0 Edge.

Self loop:→

end and beginning point is same.

Parallel / Multiple Edges:→



- Two or more edges with the same set of end points are said to ||al edges.

Simple Graph

- No multiple edges & no self loop.

multigraph:→

- Multiple edges and Self loop.

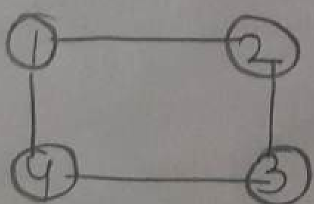
Undirected graph.

⇒ No direction graph.

Directed Graph. / Digraph:→

Direction given.

e.g. let $V = \{1, 2, 3, 4\}$ and $E = \{(1,2), (1,4), (3,4), (2,3)\}$



degree

$d(1) = 2$

$d(2) = 2$

$d(3) = 2$

$d(4) = 2$

degree for Self loop count as = 2

Degree of vertex :→

- Degree of vertex is the no. of edge incident on a vertex V.
- The Self loop is counted twice.
- It is denoted by d(V).

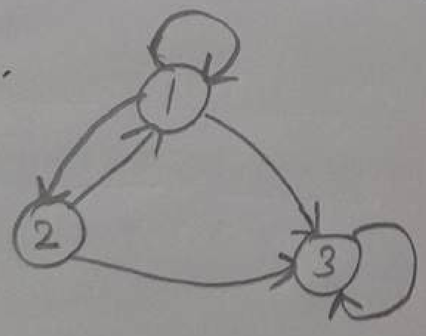
Indegree :-

- The Indegree of a vertex V in a directed graph is the no. of edges ending at a vertex V.
- It is denoted by indeg(V).

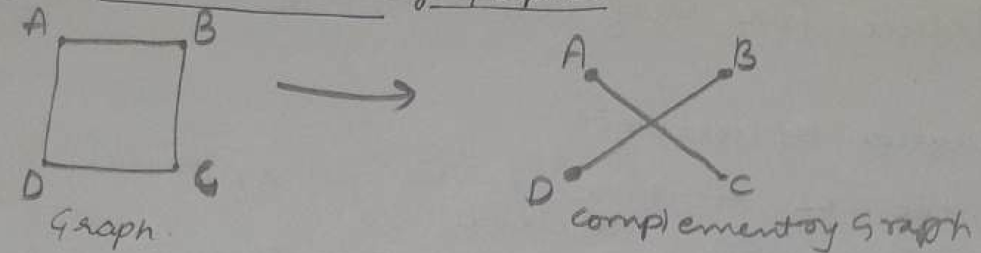
Outdegree :→

- Outdegree is the no. of edges beginning at vertex V.
- It is denoted by outdeg(V).

e.g.



Complementary Graph:→



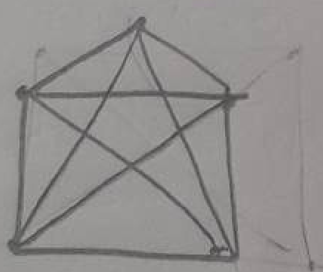
The compliment $\bar{G}$ of the Graph G is defined to be a graph which has the same no. of vertices as in Graph G and has two vertices connected if and only iff they are not connected in graph G.

Complete Graph:→

$$K_n = \frac{n(n-1)}{2} \text{ edges}$$

denoted as

$$K_5 = \frac{5(\overset{2}{\cancel{4}})}{\cancel{2}} = 10 \text{ edges}$$



- A complete Graph G on n vertices is a graph in which each vertex is connected to every other vertex.
- A complete Graph with n vertices is denoted by K_n and it will have

$\frac{n(n-1)}{2}$ edges

Regular Graph | K-regular graph.

K = Degree of vertex.

- A Graph G is regular if every vertex has the same degree.

e.g.

2-Regular graph.

Biparite Graph:→

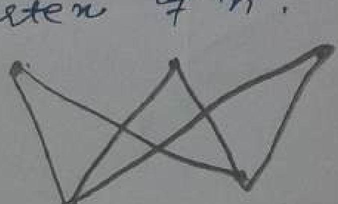
- If the vertex V can be Partition into Subset m and n such that each edge of G connects a vertex of m to a vertex n

e.g. G → 5 vertex

$5 \to 2, 3$

complete Bi-parite Graph:→

- If each vertex of m connects to each vertex of n.



Subgraph :-

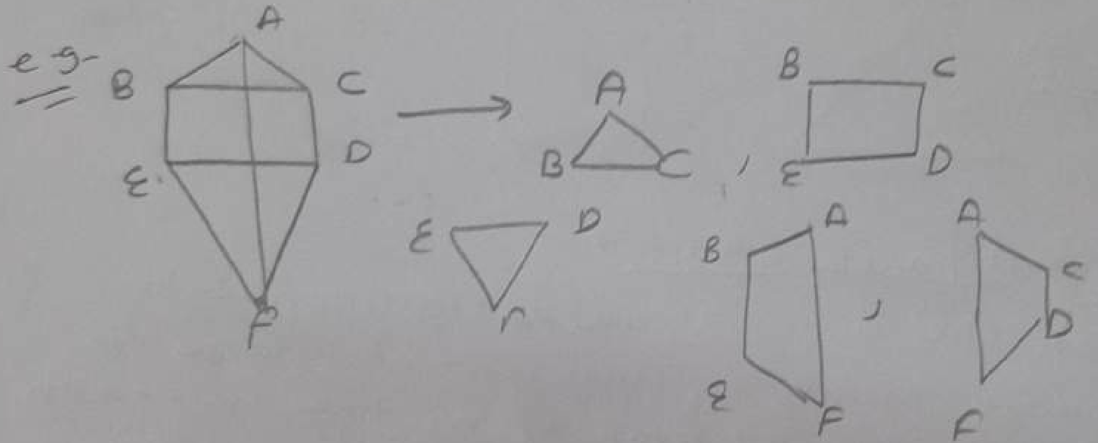
e.g. $G = G(V, E)$

Subgraph — $H = H(V', E')$

then if $V' \subseteq V$, $E' \subseteq E$

and each edge of H has the same end vertex in H as in Graph G.

e.g-

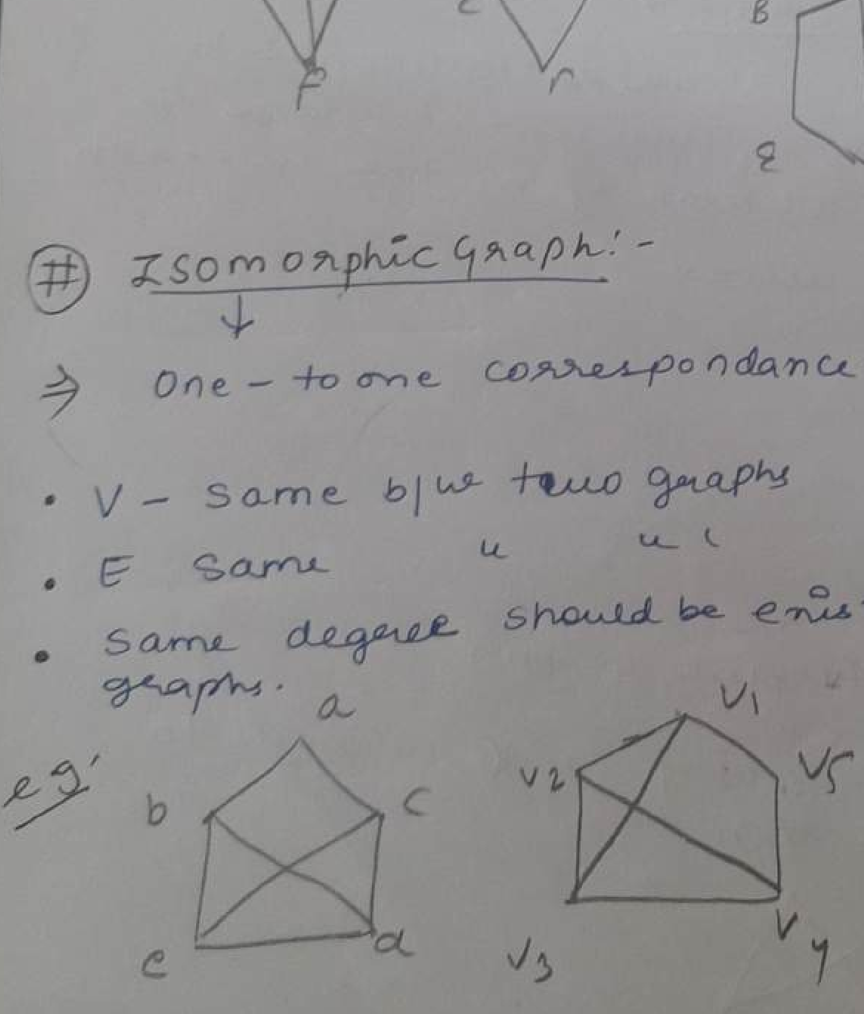


Isomorphic Graph :-

⇒ One – to one correspondance b/w vertex & Edges.

- V – same b/w two graphs
- E same " "
- Same degree should be exist for both the graphs.

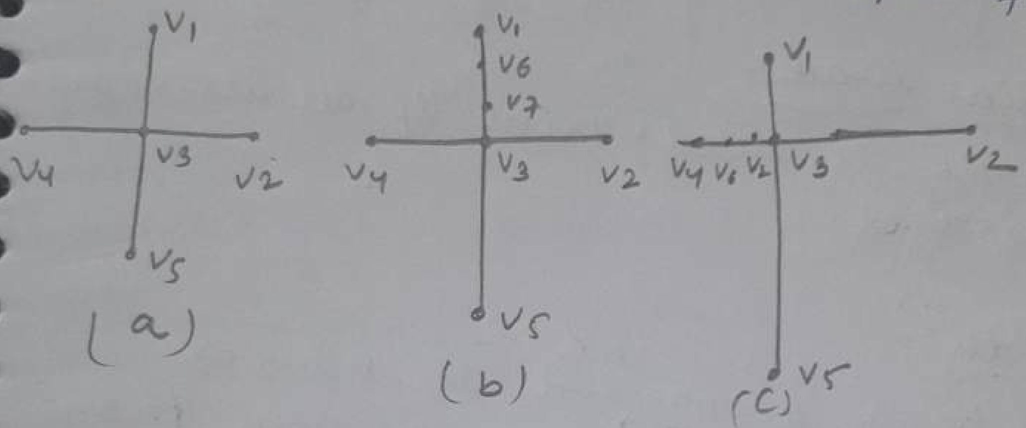
e.g.



Homomorphic graph:-

Two Graphs G and G' are said to be Homomorphic if graph G' can be obtained from G with additional vertex.

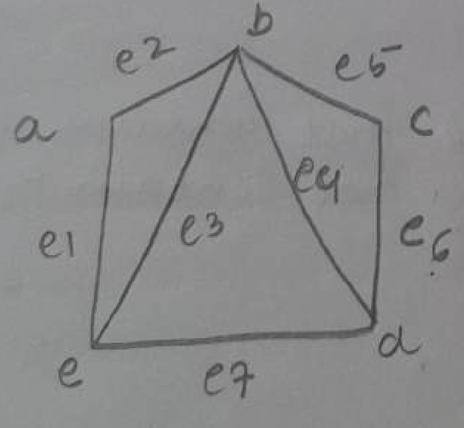
e.g. The graph b & C are Homomorphic of Graph A.



walk.

let G be a graph V and w be vertices in G. A walk from v to w is finite alternating sequence of vertices & edges of G.

e.g.



Trival Walk :→

• The trival walk from V to V consisting of single ~~verticx~~ V.

Trial walk :-

• A walk is called a Trial if all its edges are distinct.

Path :→

Let G be a graph and let V and W be vertices in G then a path from V to W is a trial that does not contain a repeated vertex.

Closed walk :→

• A closed walk is a walk that starts and ends at the same vertex.

Circuit :→

• A ckt is a closed walk that contains atleast one edge and does not contain a repeated edge.

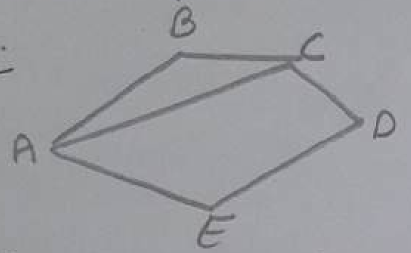
Cycle:→

• A simple ckt is a ckt that does not have any repeated vertex except first and last

Connected Graph:-

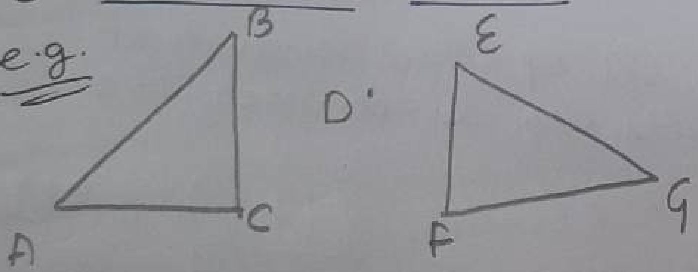
• G is connected if and only if for every vertices V(W) belonging to V(G). There exist a walk from v to w.

e.g.



Disconnected Graph:

e.g.



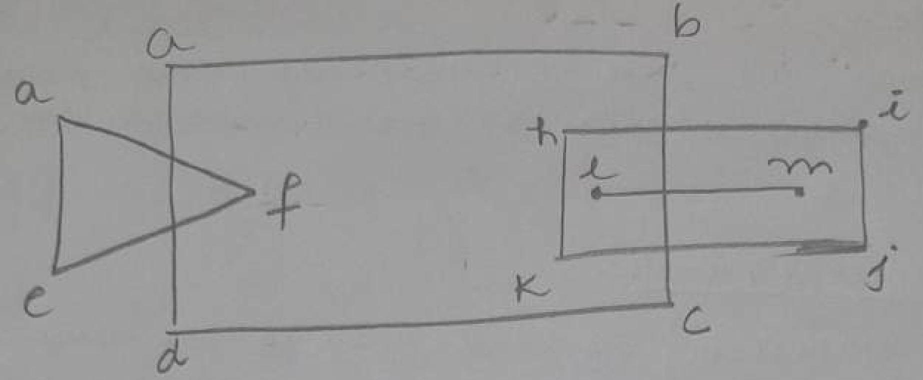
Connected component:-

• A Graph H is a connected component of a graph G if and only if

(i) H is Subgraph of G.

(ii) H is connected

(iii) No connected subgraph of G has ~~edges~~ H as a subgraph and contains vertices or edges that are not in edge.



G has 4 connected components.

(a,b,c,d) , (g,e,f)

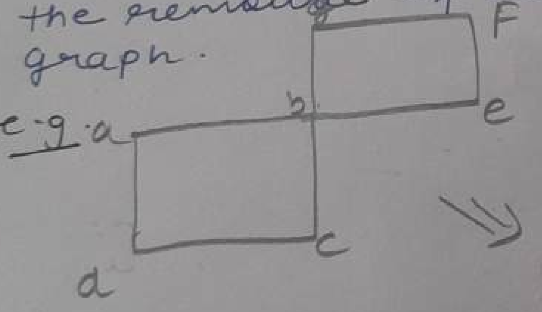
, (h, i, j, k)

, (l,m)

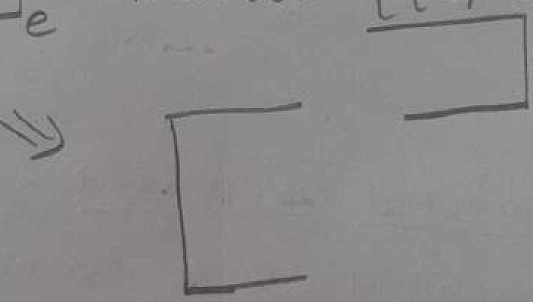
Cut Sets :->

• It is the smallest set of edges such that ~~the~~ disconnect the the removage of sets graph.

e.g.



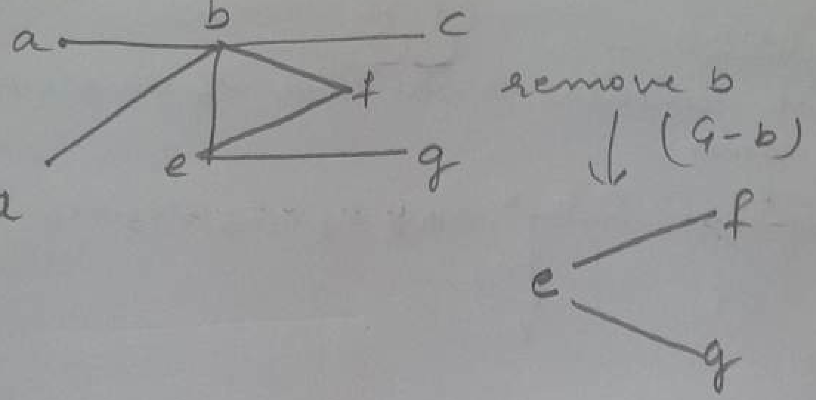
Remove {(b,C),(g,b)}



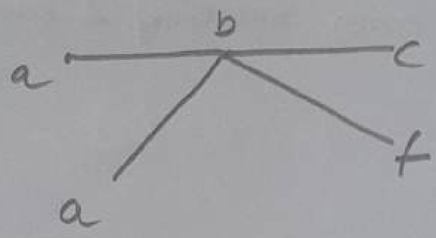
cut points:-

• A cut point for a connected G is a vertex v such that G be has more connected components than G or it is disconnected.

e.g.



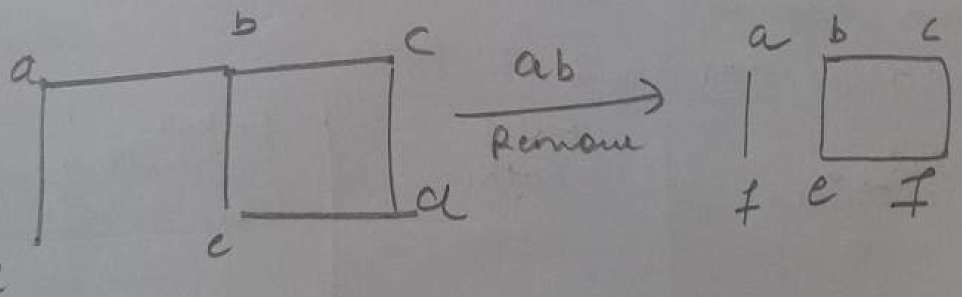
(G-e)



⊕ cut edges | Bridge :-

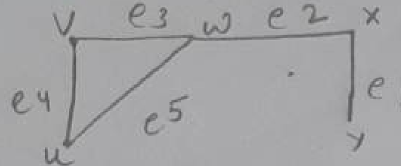
• A bridge for a connected graph G is an edge E such that G(E) has more connected components than G or disconnected.

e.g.



Euler path:-

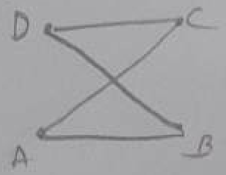
- Edge list contains each edge of the graph exactly ones.



Path:- $y\, e_1\, x\, e_2\, w\, e_5\, u\, e_4\, v\, e_3\, w$

Euler circuit:-

Edges exactly ones. Starting & ending vertex same.



Euler Graph:-

A graph that possess Euler ckt

Even degree = ckt

Zero or exactly 2 = Path

Odd vertices	Euler Path	Euler circuit
0	yes	yes
2	yes	No
4, 6, 5 ---	NO	No