Delta

03/02/2020 MATHS

Principle of Mathematical Induction.

Ques : $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

for P(1)

L.H.S. $= 1^2 = 1$

R.H.S. $= \frac{1(1+1)(2+1)}{6} = 1$

L.H.S. = R.H.S.

P(1) is true

Assume P(k) is true.

$P(k): 1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$

Now we have to prove P(k+1) is true

$P(k+1): 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

L.H.S. $\rightarrow \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{(k+1)}{6}\left[2k^2 + k + 6k + 6\right]$

$= \frac{(k+1)}{6}\left[2k^2 + 7k + 6\right]$

$= \frac{(k+1)}{6}\left[2k^2 + 4k + 3k + 6\right]$

$= \frac{1}{6}(k+1)(k+2)(2k+3)$

= R.H.S.

Hence P(k+1) is also true, $\therefore$ P(n) is true by PMI.

Dt.:
Pg.:

Ques: $7^n - 3^n$ is divisible by 4

$P(1)$: $7^1 - 3^1$
$= 4$
which is divisible by 4

Assume $P(k)$ is true
$P(k)$: $7^k - 3^k$ is div. by 4
i.e., $7^k - 3^k = 4m$

Now we have to prove $P(k+1)$ is true
$P(k+1)$: $7^{k+1} - 3^{k+1}$ is div. by 4
$= 7^k \cdot 7 - 3^k \cdot 3$
$= [4m + 3^k]7 - 3^k \cdot 3$
$= 28m + 4 \cdot 3^k$
$= 4[7m + 3^k]$
which is div. by 4
$P(k+1)$ is true $\therefore$ $P(n)$ is true by PMI.

Ques! $1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}$

for $P(1)$:
L.H.S. $= 1$
R.H.S. $= 1/3$
$\therefore$ L.H.S. $>$ R.H.S.
$P(1)$ is true.

assume $P(k)$ is true
$P(k)$! $1^2 + 2^2 + 3^2 + \dots + k^2 > \frac{k^3}{3}$

Now we have to prove $P(k+1)$ is true.

$P(k+1): 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 > \frac{(k+1)^3}{3}$

$$\text{L.H.S.} > \frac{k^3}{3} + (k+1)^2 \qquad \cancel{\frac{(k+1)^2(k-2)}{3}}$$

$$> \frac{k^3}{3} + k^2 + 2k + 1$$

$$> \frac{k^3 + 3k^2 + 6k + 3}{3} \qquad \frac{k^3 + 1 + 3k^2 + 3k}{3}$$

$$> \frac{k^3 + 3k^2 + 3k + 1}{3} + \frac{3k+2}{3}$$

$$> \frac{(k+1)^3}{3} + \frac{3k+2}{3}$$

$$> \frac{(k+1)^3}{3}$$

L.H.S. > R.H.S.

P(k+1) is true, P(n) is true by PMI.

Well oddering Principle

Every non-empty set of natural numbers has a smallest element.

\# Recursive defination of a f^n defines the values of a f^n for inputs in terms of the value of same f^n for another input.

Fibonacci → $(n+1)! = (n+1) \times n!$

Factorial → $F_1 = F_2 = 1$, $F_{n+2} = F_{n+1} + F_n$

Dt.: 4/2/2020

Division Algorithm
Given natural number a & b, there are unique non-negative integers Q & R such that
$$a = bQ + R \quad , \quad R \geq 0 \ \& \ R < b$$

Greatest common divisor algo
Largest integer that divides each of given integer such that their remainder is 0, it is

Fundamental Thm of Airthmatic
Any Integer greater than 1 is either a prime no. or can be written as a unique product of prime no.s.

05|02|2020

MATHS DISCRETE

MODULE-2

Counting techniques

- Addition Rule OR $n+m$
- Multiplication Rule AND $n \times m$

Topics

1. Inclusion - Exclusion
2. Pigeon-hole principle
3. Permutation & Combination

Inclusion & Exclusion Principle

Sets : $A_1, A_2, A_3, A_4, \ldots A_n$

$$|A_1 \cup A_2 \cup \ldots A_n| = \sum_{i=1}^{n} |A_i| - \sum_{1 \le i < j}^{n} |A_i \cap A_j| + \sum_{1 \le i < j < k}^{n} |A_i \cap A_j \cap A_k| - \ldots + (-1)^{n+1} |A_1 \cap A_2 \cap A_3 \ldots \cap A_n|$$

Proof: If A is an element that occurs in exactly R numbers of sets form A_1 to A_n then,

R is counted $C(r,1)$ times by $\Sigma |A_i|$,
it is counted $C(r,2)$ times by $\Sigma |A_i \cap A_j|$,
it is counted $C(r,3)$ times by $\Sigma |A_i \cap A_j \cap A_k|$,
this continues till $C(r,r)$.

So the element A in the inclusion - exclusion formula is counted $C(r,1) - C(r,2) + C(r,3) + \ldots \pm C(r,r)$ times.

By binomial theorem, we know that

$$\sum_{k=0}^{n} (-1)^k C(n,k) = 0$$

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On expanding above result & replace n by r

$$C(r,0) - C(r,1) + C(r,2) - \dots \pm C(r,r) = 0$$

$$C(r,0) = C(r,1) - C(r,2) + C(r,3) \dots \pm C(r,r)$$

$$1 = C(r,1) - C(r,2) + C(r,3) \dots \pm C(r,r)$$

So, term A is occuring exactly once

Pigeon Hole Principle

This principle states that if there are k+1 or more pigeons placed into k pigeon holes than atleast one hole must contain two or more pigeons.

The extended version of this principle states that if k objects are placed in n boxes, than atleast one box must hold atleast ceilvalues of $\left\lceil \frac{K}{n} \right\rceil$ objects

10/02/2020

PROPOSITIONAL LOGIC [MODULE - 3]

Basic operators
AND, OR, NOT

Derieved operators
NAND, NOR, XOR

Conditional operators
(If statement 1 than statement 2)

Logical implication $p \rightarrow q$
If p then q

hypothesis, premisses, antecedent ← p

$\boxed{p \rightarrow q \cong \sim p \vee q}$

q → conclusion, consequence

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Forms
1. Statement $p \rightarrow q$
2. converse $q \rightarrow p$
3. Inverse $\sim p \rightarrow \sim q$
4. contrapositive $\sim q \rightarrow \sim p$

Ques : The home team win, whenever it is raining.

Sol :
p : It is raining
q : Home team wins

Biconditional operator

$p \longleftrightarrow q$

q is true iff p is true

$p \longleftrightarrow q \cong p \to q \wedge q \to p$

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of logic operators

Priority

1. $\sim, \neg, '$ (NOT)
2. $\wedge$ (and)
3. $\vee$ and $\oplus$ (OR & XOR)
4. $\longrightarrow$ (conidi tional)
5. $\longleftrightarrow$ (bi conditional)

Ques : $(p \to q \wedge \sim r) \longleftrightarrow r \oplus q)$

p	q	r	$\sim r$	$q \wedge \sim r$	$p \to q \wedge \sim r$	$r \oplus q$	final
T	T	T	F	F	F	F	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	F
T	F	F	T	F	F	F	T
F	T	T	F	F	T	F	F
F	T	F	T	T	T	T	T
F	F	T	F	F	T	T	T
F	F	F	T	F	T	F	F

Ques ÷ $q \wedge r \to p \wedge \sim q \vee p \to r$

Tautology / Contradiction / Contingency

Tautology	Contradiction	Contingency
• Always True	Always False	Sometimes True, Sometimes False
• Validity	×	×
• Satisfiability	×	✓

Laws of Logic

(i) commutative

$p \wedge q \equiv q \wedge p$ $\qquad$ $p \vee q \equiv q \vee p$

(ii) Associative

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $\qquad$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(iii) Distribution

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(iv) Identity

$p \vee F = p$ $\qquad$ $p \wedge F = F$

$p \wedge T = p$ $\qquad$ $p \vee T = T$

(v) Negation

$p \wedge \sim p \equiv F$ $\qquad$ $p \vee \sim p \equiv T$

Dt.:
Pg.:

(vi) Idempotent

$p \wedge p \equiv p$ $\quad\quad$ $p \vee p \equiv p$

(vii) Absorption

$p \wedge (p \vee q) \equiv p$ $\quad\quad$ $p \vee (p \wedge q) \equiv p$

(viii) De-Morgan's Law

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

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MATHS

Argument

An argument is an insertion that given a group of proposition called premises, it yields another proposition called the conclusion.

A valid argument is an argument in which conclusion is true whenever all the premises are true. The argument is also valid if and only if the and operation on the group of proposition implies the conclusion is a tautology.

Rules & Their validity

1. Modus Ponen

$P \to Q$ is true

P is true

$\therefore Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

2. Modus Tollens

$P \to Q$

$\sim Q$

P	Q	$P \to Q$	$\sim Q$	$\sim P$
T	T	T	F	F
T	F	F	T	F

(5.) **Constructive Dilemma**

$$\begin{array}{l} (p \to q) \wedge (r \to s) \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

(6.) **Absorption**

$$\begin{array}{ll} p \to q & T \\ \hline \therefore p \to (p \wedge q) & T \end{array}$$

(7.) **Simplification**

$$\begin{array}{ll} p \wedge q & T \\ \hline \therefore p & T \\ \therefore q & T \end{array}$$

(8.) **Conjunction**

$$\begin{array}{ll} p & T \\ q & T \\ \hline \therefore p \wedge q & T \end{array}$$

(9.) **Disjunction**

$$\begin{array}{ll} p & T \\ \hline \therefore p \vee q & T \end{array}$$

Ques: Prove the validity of the following argument
- "Either Ram is a good boy or Rahul is a good boy"
- "Ram is not a good boy therefore Rahul is a good boy".

Ques: Show the hypothesis:
- "It is not sunny this afternoon & it is colder than yesterday"
- "we will go swimming only if it sunny"
- "If we don't go swimming then we will go hiking trip"
- "If we go for hiking trip then we will be home by sunset"

Show that it leads to conclusion that we will be home by sunnet.

Sol:

p: It is sunny
q: It is colder than yesterday
r: We go for swimming
s: We go for hiking
t: We come home by sunset

$$\begin{array}{ll} \sim p \wedge q & T \\ p \to r & T \\ \sim r \to s & T \\ s \to t & T \\ \hline \therefore t & \end{array}$$

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MATHS

Predicates & Quantifiers

- Universal ($\forall$)
- Existential ($\exists$)

$\forall(x).\ P(x)$	All ✓	$\sim\boxed{\forall(x)\ P(x)} \equiv \exists(x).\ \sim P($
$\forall(x)\ \sim P(x)$	All ×	
$\exists(x)\ P(x)$	Some ✓	$\sim\boxed{\exists(x) \cdot P(x)} \equiv \forall(x).\ \sim P($
$\exists(x)\ \sim P(x)$	Some ×	