December 2023

B. Tech (IT/CE (Hindi Medium)/CE/CSE/CSE (AIML)) – III SEMESTER

Mathematics III (Calculus and Ordinary Differential Equations)(BSC-301)

Time: 3 Hours **Max. Marks: 75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 mark each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Write the type of the sequence {-1, 1, -1, 1,...}. Is it convergent? (1.5)

(b) What is positive term series? (1.5)

(c) Test $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$ exists or not. (1.5)

(d) If $u = (x-y)(y-z)(z-x)$, then find $\frac{\partial u}{\partial y}$. (1.5)

(e) Evaluate $\int_0^1 \int_0^1 x\, e^y\, dy\, dx$. (1.5)

(f) State Green's theorem. (1.5)

(g) Find the integrating factor for the differential equation (1.5)

$$2 \cos x \frac{dy}{dx} + 4 \sin x\; y = 0$$

(h) Check if the following differential equation is exact: (1.5)

$$(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$$

(i) What is Clairaut's type equation? Give an example. (1.5)

(j) Identify the nature of the singular points of the differential equation (1.5)
$x^2(x-2)y'' + (x-1)y' + 2xy = 0$

PART -B

Q2 (a) Test the convergence of $\sum_{n=1}^{\infty}(\sqrt{n^4+1}-\sqrt{n^4-1})$ (8)

(b) Using Taylor's series expansion, prove that (7)

$$\log_e(1+e^x) = \log_e 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots$$

Q3 (a) If $z = f(x, y)$ where $x = u^2 - v^2, y = 2uv$, prove that (8)

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2+v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$

(b) Find the minimum value of the function $x^2 + y^2 + z^2$ subject to the condition $xy + yz + zx = 3a^2$. (7)

Q4 (a) Using Gauss divergence theorem, evaluate $\iint_S \bar{F}.\bar{n}\, dS$ where $\bar{F} = 4xz\bar{\imath} - y^2\bar{\jmath} + yz\bar{k}$ and S is the surface of the cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$. (8)

(b) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate. (7)

Q5 (a) Solve the differential equation $p^2 - p(e^x + e^{-x}) + 1 = 0$ where p has usual meaning. (8)

(b) Solve $(2x + y + 1)dy = (x + y + 1)dx$. (7)

Q6 (a) Solve the following differential equation by using variation of parameter $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$. (8)

(b) Find the power series solution of $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ in powers of x. (7)

Q7 (a) Solve $(D^4 - 1)y = e^x \cos x$. (8)

(b) Find the directional derivative of $2yz + z^2$ in the direction of the vector $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$ at the point (1, -1, 3). (7)
