Roll No. 17001.008008 Total Pages : 4

305304

Dec., 2018
B.Sc. Ist Semester
MATHEMATICS III
(BS-301)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Define trigonometric polynomial. (1.5)

(b) Find the Inverse Laplace transform of $\frac{1}{(s+a)^2}$. (1.5)

(c) If $L\{f(t)\} = F(s)$ then show that $L\left\{\frac{1}{t} f(t)\right\} = \int_s^\infty F(s)\, ds$, provided the integral exist. (1.5)

(d) State and prove change of scale property of Fourier transform. (1.5)

(e) Find the Fourier Transform of

$$f(x)=\begin{cases}1 & \text{for } |x|<1\\ 0 & \text{for } |x|>1\end{cases} \quad (1.5)$$

(f) Show that $z\left(\frac{1}{n}\right)=z\log\left(\frac{z}{z-1}\right)$. (1.5)

(g) State Convolution theorem in Z-transform. (1.5)

(h) State Stoke's theorem. (1.5)

(i) Prove that $\text{div}\left(\frac{\vec{r}}{r^3}\right)=0$. (1.5)

(j) If $\vec{F}=(5xy-6x^2)\hat{i}+(2y-4x)\hat{j}$, evaluate $\int_C \vec{F}\cdot dr$ along the curve C in the xy-plane $y = x^3$ from the point (1,1) to (2,8). (1.5)

PART–B

2. (a) Evaluate $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}dt$, by Laplace transform. (7)

(b) Prove that $\int_{-1}^{1}\frac{T_m(x)\,T_n(x)}{\sqrt{1-x^2}}dx=\begin{cases}0, & m\neq n\\ \frac{\pi}{2}, & m=n\neq 0\\ \pi, & m=n=0\end{cases}$

where $T_n(x)$ is the Chebyshev's polynomial. (8)

3. (a) Apply Convolution theorem to evaluate

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}. \quad (7)$$

(b) Solve $ty''+y'+4ty=0$, where $y(0)=3$, $y'(0)=0$. (8)

4. (a) Find the Fourier Cosine transform of e^{-ax}, hence evaluate $\int_0^\infty \frac{\cos\lambda x}{x^2+a^2}dx$. (7)

(b) Using finite Fourier transform, solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, given that $u(0, t) = 0$, and $u(x, 0) = 2x$ when $0 < x < 4$, $t > 0$. (8)

5. (a) Find the Inverse Z-transform of $\frac{5z}{(2-z)(3z-1)}$. (7)

(b) Solve $y_{n+2}-6y_{n+1}+8y_n=2^n+6n$ by Z-transform. (8)