6. (a) Evaluate $\iint_S \bar{F}\cdot\hat{n}\,ds$, where $\bar{F}=xy\hat{i}-x^2\hat{j}+(x+z)\hat{k}$ and S is the region of the plane $2x+2y+z=6$ in the first octant. (7)

(b) Verify Green's theorem to evaluate the line integral $\int_C (2y^2dx+3x\,dy)$ where C is the boundary of the closed region bounded by $y=x$ and $y=x^2$. (8)

7. (a) Find the directional derivative of V^2, where $\bar{V}=xy^2\hat{i}+zy^2\hat{j}+xz^2\hat{k}$ at the point (2, 0, 3) in the direction of the outward normal to the sphere $x^2+y^2+z^2=14$ at the point (3, 2, 1). (8)

(b) If $\bar{a}$ is a constant vector show that

$$\bar{a}\times(\bar{\nabla}\times\bar{r})=\bar{\nabla}(\bar{a}\cdot\bar{r})-(\bar{a}\cdot\bar{\nabla})\bar{r}.$$ (7)

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305304

December, 2019

B.Tech. (ECE/EIC) - III SEMESTER

Mathematics-III (BS301)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART - A

1. (a) Express $f(x)=1+2x-3x^2+4x^3$ in terms of Legendre polynomial. (1.5)

(b) State and prove shifting property of Fourier transform. (1.5)

(c) Express $f(x)=16x^4+4x^3+2x^2+4x+5$ in terms of Chebyshev polynomial of first kind. (1.5)

(d) Find the Fourier transform of $\frac{1}{x}$. (1.5)

(e) Find the Z-transform of $(k) = \frac{1}{(k+1)}, k \geq 0$. (1.5)

(f) Find the Laplace transform of $(t) = \sin at - at \cos at$. (1.5)

(g) Define convolution theorem for Laplace transform. (1.5)

(h) Find $\nabla \cdot \nabla \phi$, where $\phi = 6x^3y^2z$. (1.5)

(i) Show that the vector field defined by

$\vec{F} = (\sin y + z)\hat{i} + x(\cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational. (1.5)

(j) Evaluate the line integral

$$\int_C [(x^2 + xy)dx + (x^2 + y^2)dy],$$

where C is the square formed by the lines $x = \pm 1$, $y = \pm 1$. (1.5)

PART - B

2. (a) Prove that $\int_{-1}^{+1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2 - 1}$. (7)

(b) Prove that $\int_0^{\infty} \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a + b)}$. (8)

3. (a) Find the inverse Laplace transform of $\log \frac{s^2 + 1}{s(s+1)}$. (7)

(b) Solve the equation $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 2\cos 3t - 11\sin 3t$ given that $y(0) = 0$ and $y'(0) = 6$ by transform method. (8)

4. (a) Find the Fourier transform of the function $f(x)$ defined as

$$f(x) = \begin{cases} 1 + \frac{x}{a}, & \text{for } -a < x < 0 \\ 1 - \frac{x}{a}, & \text{for } 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$ (8)

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 6, t > 0$ subject to conditions

$u_x(0, t) = u_x(6, t) = 0$, $u(x, 0) = 2x$. (7)

5. (a) Find the Z-transform of $c^k \cos ak, k \geq 0$. (7)

(b) Find the inverse Z-transform of the function

$f(z) = \frac{3z^2 + 2z}{z^2 - 3z + 2}$ for $1 < |z| < 2$. (8)