2100101S022

Sr. no. 015301

January 2023
B. Tech 3rd Semester (ENC/ECE/EEIOT)
Mathematics - III (BS301)

Time: 3 Hours **Max. Marks: 75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part - B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

Q1 (a) Find $L(5)$, where L denotes Laplace trasform. (1.5)

(b) Write shifting property of Fourier transform. (1.5)

(c) Describe trigonometric polynomial. (1.5)

(d) What is the value of $T_1(x)$, where $T_n(x)$ is the Chebyshev polynomial of first kind of order n? (1.5)

(e) Give two examples of orthogonal polynomial. (1.5)

(f) Find $Z(a^n)$, where Z denotes Z transform. (1.5)

(g) Find $\nabla\phi$, if $\phi = \log(x^2 + y^2 + z^2)$. (1.5)

(h) State Stoke's theorem. (1.5)

(i) Define irrotational vector. (1.5)

(j) What is the greatest rate of increase of $u = xyz^2$ at the point (1,0,3)? (1.5)

PART-B

Q2 (a) Find the Laplace transform of $(1 + te^{-t})^3$. (7)

(b) Find the inverse Laplace transform of $\log \frac{s+1}{s-1}$. (8)

Q3 (a) Evaluate $\int_0^\infty te^{-3t} \sin t dt$ by using Laplace transform. (7)

(b) Solve the following differential equation by Laplace transform method (8)

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$$

when $x = 2$ and $\frac{dx}{dt} = -1$ at $t = 0$.

Q4 (a) Find the Fourier transform of (7)

$$f(x) = \begin{cases} 1-x^2, |x| \le 1, \\ 0, \quad |x| > 1 \end{cases}$$

(b) Find the Fourier cosine transform of e^{-x^2} and hence evaluate Fourier sine transform of xe^{-x^2}. (8)

Q5 (a) Find the Z transform of $3n - 4\sin\frac{n\pi}{4} + 5a$. (7)

(b) Find the inverse Z transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. (8)

Q6 (a) A vector field is given by $\mathbf{F} = \sin y\,\hat{\imath} + x(1+\cos y)\,\hat{\jmath}$. Evaluate the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$. (7)

(b) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$, where C is bounded by $y = x$ and $y = x^2$. (8)

Q7 (a) Evaluate curl $\mathbf{F}$ at the point (1,2,3), given $\mathbf{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$. (7)

(b) Evaluate $\int \mathbf{F}.ds$ where $\mathbf{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ and S is the surface bounded by the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$, by applying Divergence theorem. (8)