6. (a) Evaluate $\int_S \vec{F}.\hat{n}\, ds$, where $\vec{F} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ and 's' is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $x = 2$, $y = 0$ and $z = 0$. 8

(b) Verify Gauss divergence theorem for the function $\vec{F} = y\hat{i} + x\hat{j} + z^2\hat{k}$ over the cylinder region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = h$. 7

7. (a) Verify Green's Theorem in the plane for :

$$\oint_c \left(2xy - x^2\right) dx + \left(x^2 + y^2\right) dy$$

where c is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$. 8

(b) Apply Stoke's theorem to evaluate :

$$\int_c \left[(x+y)\,dx + (2x-z)\,dy + (y+z)\,dz\right]$$

where c is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6). 7

Roll No. 23001017055

Total Pages : 04

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December 2024

B. Tech. (ECE/EE/ENC) (Third Semester)

Mathematics-III (BS-301)

Time : 3 Hours] [*Maximum Marks : 75*

Note : It is compulsory to answer all the questions (1.5 marks each) of Part A in short. Answer any *four* questions from Part B in detail. Different sub-parts of a question are to be attempted adjacent to each other.

Part A

1. (a) State and prove Linearity Property of Laplace-Transform. 1.5

(b) Find the Laplace Transform of the function $f(t) = |t-1| + |t+1|, t \geq 0$. 1.5

(c) If $f(t)$ is a periodic function with period T, i.e., $f(t+T) = f(t)$, then prove that : 1.5

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\, dt}{1 - e^{-sT}}$$

(d) State and prove shifting property for Fourier Transform. 1.5

(e) Define finite Fourier sine and cosine Transform. 1.5

(f) Find the Z-Transform of $e^t \sin t$. 1.5

(g) Find the Z-Transform of $n \sin n\theta$. 1.5

(h) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where 't' is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. 1.5

(i) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2, –1). 1.5

(j) Show that vector field defined by :

$$\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$$

is irrotational. 1.5×10=15

Part B

2. (a) Using Laplace Transform, evaluates : 8

$$\int_0^t \frac{e^t \sin t}{t} dt$$

(b) Find the inverse Laplace Transform of : 7

$$\frac{s}{(s^2+1)(s^2+4)(s^2+9)}$$

3. (a) Using Laplace Transform, solve : 8

$$(D^2+9)x = \cos 2t, \text{ if } x(0) = 1, x(\pi/2) = -1, D = d/dt$$

(b) Solve the given PDE $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = x$, $x > 0$, $t > 0$ with boundary and initial condition $u(0, t) = 0$, $t > 0$ and $u(x, 0) = 0$, $x > 0$ using Laplace Transform. 7

4. (a) Find the Fourier cosine transform of $f(x) = 1/(1+x^2)$. 8

(b) Solve :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

given that $u(0, t) = 0$, $u(4, t) = 0$, $u(x, 0) = 2x$, when $0 < x < 4$, $t > 0$, using Finite Fourier transform. 7

5. (a) Find the inverse Z-Transform of : 8

$$\frac{z^3 - 20z}{(z-2)^3(z-4)}$$

(b) Using Z-Transform, solve $u_{n+2} - 2u_{n+1} + u_n = 3n+5$. 7