013306

December 2023
B.Tech. (Mechanical Engineering)- IIISEMESTER
MATHEMATICS III (BSC 201)

Time: 3 Hours   **Max. Marks:75**

Instructions: 1.   It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.

2.   Answer any four questions from Part -B in detail.

3.   Different sub-parts of a question are to be attempted adjacent to each other.

PART -A

Q1 (a) Find a partial differential equation by eliminating a and b from (1.5)
$z = ax + by + a^2 + b^2$.

(b) Classify the following partial differential equation: (1.5)

$$\partial^2 z/\partial x^2 + \partial^2 z/\partial y^2.$$

(c) Explain the difference between homogenous and non-homogenous linear partial (1.5) differential equations with example.

(d) What is boundary value problems. (1.5)

(e) If X is binomially distributed with parameters n and p, what is the (1.5) distribution of $Y = n - X$.

(f) Write the limitations of regression. (1.5)

(g) Prove that if one of the regression coefficients is greater than unity then other must (1.5) be less than unity.

(h) For a distribution the mean is 10, variance is 6, γ_1 is 1 and β_2 is 4. Find the first (1.5) four moments about the origin, symbols have their usual meanings.

(i) What is the mean of the $B\left(4, \dfrac{1}{3}\right)$ binomial distribution? (1.5)

(j) Criticize the following statement: (1.5)
"The mean of a binomial distribution is 5 and standard deviation is 3."

PART -B

Q2 (a) Solve $\partial^2 z/\partial x^2 + \partial^2 z/\partial y^2 = \cos mx \cos ny$. (7)

(b) Solve $\left(D^2 - DD' + D' - 1\right)z = \cos(x+2y) + e^y$. (8)

Q3 (a) Using the method of separation of variables, solve $(\partial u/\partial x) = 2(\partial u/\partial t) + u$, where (8)
$u(x,0) = 6e^{-3x}$.

(b) Show that the general solution of wave equation $c^2\left(\partial^2 u/\partial x^2\right) = \partial^2 u/\partial t^2$ is (7)

$u(x,t) = \phi(x+ct) + \psi(x-ct)$, where ϕ and ψ are arbitrary functions.

Q4 (a) Urn A contains 2 white and 2 black balls. Urn B contains 3 white and (7)
2 black balls. One ball is transferred from A to B and then one ball is drawn
out of B. Find the chance that this ball is white. If this ball turns out to be
white, find the probability that the transferred ball was white.

(b) Derive Poisson distribution as the limiting case of Binomial (8)
distribution, where $n \to \infty$, $p \to 0$ and $np = \lambda$ (a finite constant).

Q5 (a) In partially destroyed laboratory record of an analysis of correlation data, the (8)
following results are only legible: variance of x is 9, regression lines: $8x - 10y + 66$
$= 0$ and $40x - 18y = 214$. What are

(i) means of x and y (ii) the S.D. of y (iii) coefficient of correlation between x
and y.

(b) If X is the number scored in a throw of a fair die, show that the Chebyshev's (7)
inequality gives $P\left[|X - \mu| > 2.5\right] < 0.47$, while the actual probability is zero.

Q6 (a) If θ is acute angle between two regression lines in case of two variables x and y, (8)
show that $\tan\theta = \dfrac{1-r^2}{r}\left(\dfrac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$ where the symbols have their usual meaning.

Explain the significance of the formula when $r = 0, r = \pm 1$.

(b) By the method of least squares, find the curve $y = a + bx$ that best fits the (7)
following data:

x	0	1	2	3	4
y	1	4	10	17	30

Q7   (a)   Establish the relation between moment about the mean and moment about   (7)
arbitrary point. The first four moments of a distribution about the value 4 of the
variate are -1.5, 17, -30 and 108. Calculate the first four moments (i) about mean (ii)
about origin (iii) about $x = 2$ and also find β_1 and β_2.

(b) A die is thrown 270 times and the results of these throws are given below: (8)

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not.

(Tabulated value of CHI-SQUARE at 5% level of significance for 5 d.f. is 11.09).

Part A

1. **Finding Partial Differential Equation**
   - Topic: Partial Differential Equations (PDEs)
   - Concept: Elimination of arbitrary constants to derive a PDE.

2. **Classification of PDE**
   - Topic: Partial Differential Equations
   - Concept: Classifying PDEs (elliptic, parabolic, hyperbolic).

3. **Homogeneous vs. Non-Homogeneous PDEs**
   - Topic: PDEs
   - Concept: Definitions and examples of homogeneous and non-homogeneous equations.

4. **Boundary Value Problems**
   - Topic: PDE Applications
   - Concept: Definition and significance in solving physical problems.

5. **Binomial Distribution Transformation**
   - Topic: Probability Distributions
   - Concept: Distribution of $Y = n - X$ when X is binomial.

6. **Limitations of Regression**
   - Topic: Regression Analysis
   - Concept: Challenges and boundaries in regression modeling.

7. **Relation Between Regression Coefficients**
   - Topic: Regression Analysis
   - Concept: Proving that the product of regression coefficients equals the square of the correlation coefficient.

8. **Moments of a Distribution**
   - Topic: Statistical Moments
   - Concept: Calculating moments using mean and variance.

9. **Mean of a Binomial Distribution**
   - Topic: Probability Distributions
   - Concept: Calculating mean np of a binomial distribution.

10. **Critiquing Statistical Statements**
    - Topic: Probability Distributions
    - Concept: Validating or criticizing given statistical claims.

Part B

1. **Solving PDEs**
   - (a) Solving $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos(mx)\cos(ny)$.
     - Topic: PDE Solutions
     - Concept: Using specific methods to solve second-order PDEs.
   - (b) Solving $(D^2 - DD' + D - 1)z = \cos(x + 2y) + e^x$.
     - Topic: Linear Differential Equations
     - Concept: Operator method for solving linear equations.

2. **Separation of Variables in PDEs**
   - (a) Using the method of separation of variables to solve PDEs.
     - Topic: PDEs
     - Concept: Fundamental technique to reduce PDEs to simpler forms.
   - (b) Deriving the general solution of the wave equation.
     - Topic: Wave Equation
     - Concept: General solutions for physical wave problems.

3. **Probability and Distribution Derivation**
   - (a) Probability problems involving conditional probability and Bayes' theorem.
     - Topic: Probability Theory
     - Concept: Analysis using conditional probability.
   - (b) Derivation of the Poisson distribution.
     - Topic: Probability Distributions
     - Concept: Poisson as the limiting case of binomial distribution.

4. **Regression Analysis**
   - (a) Calculating means, standard deviation, and correlation coefficients from regression equations.
     - Topic: Regression and Correlation
     - Concept: Extracting parameters from regression models.
   - (b) Chebyshev's inequality and its significance.
     - Topic: Probability Inequalities
     - Concept: Comparing inequality bounds with actual probabilities.

5. **Least Squares Method**
   - (a) Fitting the curve $y = a + bx$ using least squares.
     - Topic: Statistical Data Fitting
     - Concept: Using least squares to derive best-fit equations.

6. **Moments and Distribution Testing**
   - (a) Establishing relationships between moments and recalculating moments about different points.
     - Topic: Statistical Moments
     - Concept: Moment transformations and calculations.
   - (b) Chi-square test for die fairness.
     - Topic: Hypothesis Testing
     - Concept: Using the chi-square test to determine bias in a sample.

Chi square test is also known as 'fit of goodness' or 'goodness of fit test